• Volume 8C, Number 1, February 1960

BULLETIN OF THE RESEARCH COUNCIL OF ISRAEL

Section C TECHNOLOGY

Bull. Res. Counc. of Israel. C. Techn.

Incorporating the Scientific Publications of the Technion — Israel Institute of Technology, Haifa

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SECOND ANNUAL CONFERENCE ON AVIATION AND ASTRONAUTICS FEBRUARY 9 AND 10, 1960

The Weizmann Science Press of Israel • Jerusalem • ירושלים • מוסד ויצמן לפרסומים במדעי הטבע ובטכנולוגיה בישראל

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SECOND ANNUAL CONFERENCE

ON

AVIATION AND ASTRONAUTICS

Sponsored by

ISRAEL SOCIETY OF AERONAUTICAL SCIENCES

ISRAEL ASTRONAUTICAL SOCIETY

DEPARTMENT OF AERONAUTICAL ENGINEERING, TECHNION-ISRAEL INSTITUTE OF TECHNOLOGY

DEPARTMENT OF CIVIL AVIATION, MINISTRY OF TRANSPORT AND COMMUNICATIONS

February 9 and 10, 1960

Sokolov House, Tel-Aviv

Technion City, Haifa

PROGRAM OF THE SECOND ANNUAL CONFERENCE ON AVIATION AND ASTRONAUTICS

9 February, 1960, Sokolov House, Tel-Aviv

8.00 REGISTRATION

9.00 OPENING ADDRESS: ALUF E. WEIZMANN, *Commander, Israel Air Force*

9.15 *FIRST SESSION*

AIRCRAFT OPERATIONS	*Chairman:* D. ABIR, *Technion–Israel Institute of Technology*
Considerations in Re-equipping a Commercial Airline	Y. PALGI, *EL-AL Israel Airlines Ltd.*
Development of Internal Air Transport	L. BIGON, *ARKIA, Israel Inland Airlines Ltd.*
The Operational and Economic Efficiency of the Airplane	B. ARAD AND A. HACOHEN, *Dept. of Civil Aviation and Chim-Avir Ltd., resp.*
Operations Research of the Aircraft	SGAN ALUF A. HILLEL, *Israel Air Force*

11.45 *SECOND SESSION*

PILOT TRAINING	*Chairman:* J. SINGER, *Technion–Israel Institute of Technology*
Flight Training in the Jet Age	SGAN ALUF Y. YAVNE, *Israel Air Force*
Conversion Training for Jet Powered Equipment	T. G. JONES, *EL-AL Israel Airlines Ltd.*

14.15 *THIRD SESSION*

THE JET TRANSPORT AIRPLANE	*Chairman:* D. SHIMSHONI, *National Council of Science and Applied Research*
Test Programs Behind a Commercial Jet Transport Aircraft	J. N. ROBINSON, *Boeing Airplane Co.*
Operation of the D.C. 8 in Terminal Area	R. BALDWIN, *Douglas Aircraft Co. Inc.*
Effect of Operational Parameters on Jet Transport	G. C. PRILL, *Convair*

16.15 *FOURTH SESSION*

ECONOMIC AND LEGAL ASPECTS OF AVIATION AND ASTRONAUTICS	*Chairman:* H. J. SHAFER, *Technion–Israel Institute of Technology*
The Struggle for Traffic in Aviation	D. BAR-NESS, *Dept. of Civil Aviation Ministry of Transport*
The National Airline's Place in Government Aviation Policy	M. BEN-ARI, *EL-AL Israel Airlines Ltd.*
On Some Political and Legal Questions Relative to Outer Space	M. MUSZKAT, *Tel-Aviv University*

20.30 *BANQUET*

Guest Speaker	I. BEN-AHARON, *Minister of Transport and Communications*

10 FEBRUARY, 1960, AERONAUTICAL ENGINEERING BUILDING, TECHNION CITY, HAIFA

10.30 *FIFTH SESSION*

AERONAUTICAL AND ASTRONAUTICAL RESEARCH	*Chairman:* E. D. BERGMANN, *Dept. of Research and Development, Ministry of Defence*
Cosmic Radiation Problems in Astronautics	K. SITTE, *Technion–Israel Institute of Technology*
On the Meteorology of the Planet Mars	J. NEUMANN, *The Hebrew University of Jerusalem*
Ionization Measurements in Shocked Gases	Y. MANHEIMER-TIMNAT, *Ministry of Defence*
A Practical Method for Calculation of Pressure Distribution on Airfoils in Supersonic Rotational Flow	A. KOGAN, *Technion–Israel Institute of Technology*

14.00 *SIXTH SESSION*

AERONAUTICAL AND ASTRONAUTICAL RESEARCH (Continued)	*Chairman:* E. JABOTINSKY, *Technion–Israel Institute of Technology*
High Frequency Oscillations of Supersonic Airfoils	M. HANIN, *Technion–Israel Institute of Technology*
A Hypersonic Ramjet Using a Normal Detonation Wave	M. ARENS, *Technion–Israel Institute of Technology*
Heat Transfer Stability Analysis of Solid Propellant Rocket Motors	R. SHINNAR AND SGAN ALUF M. DISHON, *Ministry of Defence and Israel Defence Forces, Ordnance Corps, resp.*
Some Developments of the End-Burning Charge	A. YARON, *Dept. of Research and Development, Ministry of Defence*

17.00 *ANNUAL MEETING OF THE ISRAEL ASTRONAUTICAL SOCIETY*

ON SOME POLITICAL AND LEGAL QUESTIONS RELATIVE TO OUTER SPACE

Marion Muszkat
Faculties of Law and Social Science, Hebrew University, Tel-Aviv Branch

The view that war as an instrument of world politics is becoming outdated because of the danger to the very existence of the entire human race with which the employment of modern armies is fraught, would appear no longer to be questioned.

Yet there are enormous political, economic, social, psychological and other difficulties that stand in the way of implementing the idea of disarmament as a means of obviating the factors most threatening to world peace. One of these is undoubtedly connected with the answer to the question recently posed by Seymour Harris in the "New York Times", viz.- "Can we prosper without arms?"

Most of the relevant findings indicate an affirmative answer to this question. The tremendous possibilities open to advanced nations, particularly the great powers (who are, incidentally, the largest producers of armaments), to assist underdeveloped countries to find their place in modern civilization, are pointed out. Moreover, it is considered no more than the bounden duty of such nations to do so, while it is noted that the satisfying of the needs involved, on a commercial basis, constitutes a gigantic, many-sided undertaking and, what is more, a not unprofitable one.

If one takes into consideration the fact that the probable cost of construction of a single manned space vehicle designed to land on the moon, for instance, and return to the earth's surface, is likely to be rather more than 20 billion dollars, one can say that the numeros categories of scientists and technicians concerned in research and industry connected with the exploration of outer space may well be proud of the fact that their work points to the existence of new possibilities of utilizing, profitably, the means at present absorbed by military budgets. This fact probably gives rise to one of the most important political and economic aspects of the manifold problems connected with outer space.

These problems without doubt contribute to the growing conviction that a) international cooperation, especially scientific and economic, ought to be continually extended if mankind is to be spared a great deal of unnecessary effort; b) the reduction of the elements which hinder such cooperation and which also constitute a danger to peace, can and should be gradually effected; and c) the recent agreement, for instance, prohibiting the use of Antartica for any military purposes whatsoever, can and must be followed by an agreement outlawing the use of outer space for military ends and

establishing international machinery for supervising the adherence to such an agreement.

It may thus be clear why, for various reasons, outer space has become the concern of the United Nations as well.

Para. I(d) or the Resolution of the General Assembly of the 13th December, 1958, adopted at the Assembly's 792nd plenary meeting gave rise to the appointment of an ad hoc committee for the Peaceful Uses of Outer Space, which decided that in order to fulfil its mandate it has 1) to select and define problems that have arisen or are likely to arise in the near future, in the carrying out of space programmes, 2) to divide the problems into two groups, those which may be emanable to early treatment and those which do not yet appear to be ripe for solution, and 3) to indicate, without definite recommendation, various means by which answers to such problems might be pursued.

It should be pointed out that the U.S.S.R. and certain other states decided to remain aloof from the work of this Committee, as they were of the opinion that the establishment of the Committee discriminated against the states of the eastern bloc. But it is apparent that it was not the struggle for enforcing the principle of equal representation of both political camps in the Committee which was the only or main reason for the U. S. S. R.'s absence from this body.

It should be borne in mind that the United Nations resolution establishing the Committee arose out of two varying approaches to the question: one, based on the assumption that attention should be paid to the use of outer space for peaceful and scientific ends only as conceived by the United States, had already found expression in the American proposal laid before the General Assembly on the 14th January 1957, and the other, supported by the U. S. S. R., pointed out the necessity for considering the problem of excluding outer space from the province of military activities as part of the general disarmament plan. At the end of 1959, however, during the 14th Session of the United Nations General Assembly, no doubt under the influence of the factors favouring the already months old relaxation of international tension, a resolution, jointly supported by all states, was adopted establishing a new United Nations outer space Commission.

This Commission will be composed of 24 members, 12 representatives of the Western bloc, 7 of the Eastern bloc, and 5 representatives of states not in either of the two political camps. It was also resolved to call a widely based international conference to discuss all problems connected with the peaceful uses of outer space. As evidence of the growing universal cooperation in this sphere, we may also mention the agreement reached in December 1959 at The Hague by the Committee on Space Research (COSPAR) established in October 1958 on the initiative of the International Council of Scientific Unions (I. C. S. U.) with a view to ensuring the cooperation of all scientific institutions dealing with research of relevant problems, and to convening international conferences to discuss questions concerning the exploration of outer space.

As regards the identification and listing of legal problems which might arise in the carrying out of programmes to explore and exploit outer space, the work done by the ad hoc Committee will most probably serve as a basis for the relevant discussions in the new United Nations Committee on this matter.

From the results of the work referred to above the following are the legal problems susceptible of priority treatment: (1) The question of the freedom of outer space for exploration and use; (2) Liability for injury or damage caused by space vehicles; (3) Allocation of radio frequencies; (4) Avoidance of interference between space vehicles, aircraft etc.

1. As regards the first question, the Committee took into account the premise of the permissibility of launching space vehicles and their flight, once launched, without protest from any state, regardless of the territory "over" which they pass during their flight through outer space, and noted the factual recognition of a rule to the effect that outer space is freely and equally available for exploration and use by all in accordance with existing practice, and that this rule might find its expression in customary law or in appropriate future agreements.

Like the freedom of the high seas, this rule may be termed the freedom of outer space, or freedom of the universe, its exploration and exploitation. But it would be of paramount importance to bring about an agreement limiting this freedom to the peaceful uses of outer space and outlawing its exlpoitation for military ends. It is thus regrettable that the Committee did not find it necessary to formulate any proposal on this subject, if one bears in mind that its terms of reference were to ensure the use of space for exclusively peaceful purposes.

2. With respect to the second question, the Committee was of the opinion that early consideration should be given to the conclusion of an agreement for the compulsory submission of disputes between states as to their liability for injury or damage caused by space vehicles, to the jurisdiction of the International Court of Justice. The Committee also pointed out that as far as liability for surface damage caused by space vehicles is concerned, the Rome Convention of 1952 on Damage Caused by Foreign Aircraft to Third Parties on the Surface and the International Civil Aviation Organisation's experience could be taken into account.

The problem no doubt involves the elaboration of international standards regarding safety and precautionary measures governing the launching and control of space vehicles. There is also some room for doubt if the experience of the I. C. A. O. as to the kinds of injury for which recovery may be had is entirely appropriate. The question, for instance, is whether there should be liability, without regard to fault, for certain or all activities, or whether liability should be based on fault. Again, should different principles apply depending on whether the injury occurs on the earth's surface, in the air space or in outer space; should liability arising out of launching be unlimited in amount, or joint or several, where more than one state or an international agency participate in any particular activity. It is just because of the differences between aircraft and space vehicles that the kind and extend of damage caused by the

latter cannot be compared with the effects of damage and injury resulting from the use of aircraft.

A possible approach to the solution of the above problems lies not only in studying analogous material based on existing arrangements, but also in coming to original conclusions as a result of a joint effort on the part of lawyers as well as experts, properly to evaluate the new phenomena and their possible consequences. Difficulties may arise, as a consequence of the Committee's proposition that disputes be compulsorily submitted to the jurisdiction of the International Court of Justice, as some states may prefer other methods of settling disputes.

3. As to the third question, it was rightly emphasized that to avoid paralyzing interference with radio transmissions, a rational allocation of frequencies for communication with and between space vehicles is imperative, and that the relevant recommendations of the international Telecommunications Union and its special committees should be taken into consideration.

4. As regards the fourth question, technical studies by interested governments with, if necessary, the assistance of competent specialized agencies, may have to be undertaken in order to prepare the basis for agreements with respect to the prevention of physical interference between space vehicles, particularly rockets, and conventional aircraft.

Questions of identification and registration of space vehicles and of the co-ordination of launchings, the return to earth and landing of space vehicles are also among the problems concerning outer space which are of primary significance.

Identification of space vehicles can be effected by reaching agreement on the allocation of individual call signs, orbital or transit characteristics and suitable markings. Such an agreement will become increasingly necessary and urgent as the number of space vehicles increases and particularly when they begin to return to earth.

There can be no doubt that identification would be facilitated by a system of registration of the launchings of space vehicles, their call signs and current orbital and transit characteristics. Registration would help to avoid the potential overloading of tracking facilities, and enable other states to distinguish between the various space vehicles registered and other objects and to take appropriate measures, if necessary, to protect their interests.

Identification, registration and co-ordination of launchings also constitute the most important elements in the solution of problems concerning property, traffic security, rescue proceedings, liability and jurisdiction.

It would seem that these problems could, in large measure, be solved by analogy with the rules governing the identification, registration and co-ordination of the various kinds of craft, and particularly with those governing navigation co-ordination and especially flights of conventional aircraft.

As regards artificial earth satelites, however, and the other types of rockets launched for exploration purposes only, it ought to be recognised that there should be no pretensions to property in these objects which should be considered as "res nullius"

-rather like a ship abandoned by its crew, a wreck, or a sea-bottle. Unmanned space vehicles and such vehicles when manned in the future can not yet be considered as "legibus solutus" in any case. It would be most appropriate, for various reasons, to bestow national status on space vehicles. This would, first of all, ensure order and resolve questions of liability for activities in space, and for the return to earth and landing of space vehicles. It may some-times become necessary for space vehicles returning to earth and landing to pass through foreign air space or land on foreign territory, whether this is occasioned by accident, mistake or distress. Such eventualities make necessary the conclusion of a multilateral agreement governing, inter alia, the means of returning space vehicles to the state which has launched them and, in the future, their personnel as well, when they are manned.

The United Nations ad hoc Outer Space Committee was of the opinion that the rules of International Law already existing with respect to aircraft and airmen landing on foreign territory through inadvertence, accident or distress, might be applied to this field as well.

According to the Committee, certain problems, such as the question of determining where outer space begins, in relation to the question of protection of public health and safety; the question of safeguarding against contamination of outer space or contamination from outer space, in relation to the exploration of celestial bodies; and the avoidance of interference of space vehicles with one another, do not appear to be ripe for solution.

As regards the first of the above problems, it may be affirmed that in accordance with existing agreements, a state's sovereign rights come to an end where the air space ends. The Roman rule "cuius est solum, eius est usque ad coelum" is, of course, no longer applicable. According to this rule, the moon might be considered as under the jurisdiction of the state in the morning and subject to that of another in the evening. There is, however, no consensus of opinion among experts as to the location of the limits of sovereignty of states in the space above the land and water over which they exercise sovereignty, and the proposition that the determination of such limits be based on the physical characteristics of the atmosphere and of aircraft, is fraught with great difficulty. Demarcation on the basis of present experience and knowledge may soon become out of date. The U. N. ad hoc Committee therefore came to the conclusion that this question be left to concretization by custom which will prevent the acceptance of a boundary so low as to interfere with existing aviation regimes or so high as unreasonably to fetter activities connected with the use and exploration of outer space.

It should, however, be borne in mind that territorial water limits became established on the basis of security needs. The 3-mile limit was established hundreds of years ago in conformity with the then-existing artillery ranges, and this limit is still preserved although today deprived of any military significance.

Taking current scientific, technical and military knowledge and experience into account, it may not be unreasonable mechanically to establish by multilateral agree-

ment the altitude limits of the exclusivity of the rights of states. This might be done in order to avoid interference which might hinder scientific exploration above the earth's surface and at the same time to regulate the use of space for military purposes, for the freedom of outer space, unlike the freedom of the high seas, should and can be limited to the right of all states of peaceful exploration and exploitation.

It is self-evident that the demarcation of outer space for the purpose of outlawing its use for military ends would constitute a valuable contribution to the advance of disarmament programmes.

If such an approach could be accepted as suitable for prompt implementation with the concomitant setting up of a supervisory body, speculation as to the need of anything similar to territorial waters or such as a contiguous zone on the border between the atmosphere and outer space, would lose all significance.

With regard to questions relating to the exploration of celestial bodies, the conventional methods of acquiring territory by declaring new-found areas to be under the sovereignty of the particular state which has discovered them etc., would appear to be outmoded, for the simple reason that the challenge of the universe can most probably be met, to a large degree, by common effort of a real international nature undertaken by international organizations in order to ensure to all nations the eventual benefits of an enterprise, being the common achievement and the expression of the common will of mankind, which will come to fruition with the advance of science and technology.

The acceptance of a rule that celestial bodies are incapable of appropriation to national sovereignty and that their exploration and exploitation be carried out exclusively for the benefit of mankind as a whole and under international administration ought to be followed by the establishment of another rule to the effect that any activities of the above nature should be so conducted as not to destroy any life phenomena on such bodies.

It is clear from what has been said above that many complex political and legal problems which have arisen as a result of the advance of astronautical science and technology demand the almost urgent creation of a new branch of law to govern them.

Whatever this law may be called - astronautical law, the law of the universe, interplanetary law, or the law of outer space - it already exists in an embryonic form and has now to be progressively but rapidly developed in order to keep up with the needs of reality and the requirements relating to the strengthening of peaceful cooperation and prevention of war.

Based in a certain measure on analogy with the provisions of maritime and air law, but at the same time having to provide original solutions for new problems, the new law must be ready to deal in the not so distant future with questions which are bound to arise when space travel is no longer a mere challenge to future generations.

COSMIC RADIATION PROBLEMS IN ASTRONAUTICS

KURT SITTE

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ABSTRACT

Recent progress in the interpretation of intensity variations of cosmic radiation has demonstrated the existence of streams of high-energy particles emitted during solar activity, and of magnetic clouds within the solar system which trap and scatter cosmic ray particles. For further study of these phenomena, research is being carried out in the Physics Department along three main lines. By operating meson detectors at sea level and underground, data on the energy distribution of the solar component are collected. By observing D-layer reflection with an ionospheric recorder—which according to theoretical considerations should be strongly affected by cosmic radiation—in correlation with solar activity, the low-energy end of the solar emission is checked. In addition, correlation of intensity variations with the observations of the solar physics laboratory is expected to yield further information about the nature of solar activity. Preliminary results of these experiments will be presented.

I. As soon as space travel became a distinct possibility for the near future, and hence the object of serious research instead of a mere dream, it also became obvious that the problems of cosmic rays in relation to astronautics, their possible effects on a space ship and its crew, had to be studied seriously. After all, we are dealing with an enormously penetrating ionizing radiation, by far exceeding in range and in complexity of its interactions any known terrestrial source, and we are well aware of the dangers of exposure even to the milder radiations of our laboratories to men and materials. Complete, or even partially efficient shielding is impossible; it was, therefore, imperative to know precisely the amount of radiation and radiation damage we have to reckon with when we venture to leave for an extended period the protection offered by the earth's atmosphere. A priori, it is quite conceivable that it might prove a prohibitive dose and hence would make manned space flight an impossibility unless or until we discover completely new ways to overcome this obstacle.

Fortunately, early investigations gave very encouraging results. When our first vehicles, balloons and rocket sonds, reached what might be called the rim of the atmosphere—that is to say, atmospheric depths of a few g/cm^2 or a fraction of a percent of the entire air layer — the total intensity of the cosmic radiation was found to be much less than what we consider in our laboratories a "permissible dose". However, two new developments have shaken this optimism in recent years, and have brought it home to us that much more work will have to be done before

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we can say with confidence that man is safe from radiation danger in space. I refer to the observations which confirmed the existences of vast regions of magnetic clouds in the galaxy, and to the discovery of the Van Allen belts[1] of radiation encircling the earth.

As to the first, it is easy to see that such clouds of ionized matter containing, and carrying along, a magnetic field will act as traps for high-energy particles. The density of radiation inside a cloud will, consequently, differ from that outside: in other words, the cosmic ray intensity is by no means quite uniform throughout the galaxy or even large sections of it, but may well show a "patchiness" of considerable degree. If that is so, how can we be certain that the intensity measured in the immediate vicinity of the earth is a true measure of the total radiation exposure expected in space flights, or even a fair average? It is not possible that we live in a sort of enclosure of low radiation density, but may encounter much higher doses once we leave its safe boundaries?

Perhaps such a conjecture seems unduly pessimistic. It is, however, not without some foundation in facts. Or better, there are some facts known to us which may well yield exactly this result, although we do not yet have a clear enough picture to give a conclusive evaluation. Recent observations of solar phenomena have demonstrated that the sun is continuously emitting streams of particles, possibly all along its equatorial region. This emission would, as it were, carry away a solar magnetic field (if the sun, like the earth, has a more or less permanent, dipole-like field). We might view it as a regular wind in that tenuous atmosphere of the sun—to which we shall have to refer still often—blowing radially with an intensity fluctuating with the solar 11-years' period, and at times erupting into short-lived hurricanes: the solar flares. In this picture, the observed slight periodical variations of cosmic ray intensities find their explanation, and likewise the occasional sudden intensity fluctuations which are as a rule accompanied by other disturbances like magnetic storms. But unless we assume that at least during part of these periods the solar wind does not reach us, that the effective range of the sun's activity ends within the earth's orbit, it also compels us to admit the serious imperfection of our knowledge of the primary cosmic ray intensity at some distance from the earth.

The discovery of the Van Allen radiation belts was even more of a shock, and for a while really seemed to make radiation damage a serious threat to space flight instead of a remotely possible danger. Perhaps one should have expected it; yet everybody was surprised when in the early satellite flights it was found that the cosmic ray intensity—supposed to be constant once we have left the earth's atmosphere behind—rather suddenly began to increase to a manifold of its supposed "saturation value", reaching amounts which would make long-term continuous exposure impossible. However, it was soon learned that this belt does not extend very far, and that beyond it the cosmic ray intensity drops back to its normal value. Or almost: for further study revealed the existence of a second Van Allen belt, still very much more densely filled with corpuscular radiation and very much wider

in its extension both in latitude and in distance. The two belts do in fact present a danger which is by no means negligible, and it is easy to predict that in the near future, a great deal of work will be done in order to elucidate origin and nature of the Van Allen belts. At present, we believe we have a consistent theory of the second, outer, radiation belt, but the views on the inner belt are still conflicting. A little more about this will be said later.

II. Turning now to the very modest contribution the Technion Physics Department can make to these efforts, I wish to describe three approaches from which we hope to obtain results of some significance. From all-too obvious reasons, we cannot indulge in the luxury of sending equipment to satellite or lunik heights; and while we may perhaps entertain reasonable expectation of flying some apparatus to moderate heights (as rocket heights go), the bulk of our equipment is earth-bound. Indeed, part of one of the experiments of astronautical interest is carried out underground, in a room just off the tunnel of the Carmelit underground railway. Let me begin with discussing this experiment.

In this investigation, μ-meson telescopes are used to extend the study of time variations of cosmic radiation over a large energy region. The connection with astrophysical problems follows from the general description of the current views on the causes of these variations given above. Without going into details, it will be clear that the various theories hitherto worked out (e.g. Morrison[2], Parker[3], Elliot[4] either predict precisely the energy dependence of the phenomena and of the degree of their correlation, or can be tested by observation of such a dependence where none, or no regularity, follows from the theoretical assumptions. This is, after all nothing more than a conclusion from the obvious fact that any magnetic trapping mechanism can operate efficiently only up to energies $E \lesssim 300\ H\rho$, where H stands for the average field intensity and ρ for the dimension of the trapping field.

In all the work hitherto reported, the energy dependence of events like flares or slow intensity decreases ("Forbush effect") has only been studied from a comparative survey of observations at different stations, i.e. on the basis of the geomagnetical latitude effect. This puts an upper limit of about 15 GeV to the primary energy involved, since particles of higher energies have too high a magnetic rigidity to be affected by the earth's field. It would, however, be of importance to learn about the presence or absence of a solar component at energies above 15 GeV, and about the possibility of trapping such particles. In order to do this, we have constructed telescopes of different opening angle, and have put one such telescope underground.

(Incidentally, the use of the word "telescope", long accepted in cosmic ray work, will seem appropriate also to others, now that astronomy with invisible radiation has become so important a part of this science. We are now beginning to learn to decode all kinds of messages sent from the stars, not only those emitted in visible light, and radio or cosmic ray telescopes take their place along with the "classical" optical instrument).

The principle of the experiment is demonstrated in Figure 1 which shows a schematic drawing of the two basic units in use. Each consists of four trays of counters, and is subdivided so that both threefold coincidences of the lower trays, and fourfold coincidences of the entire system are continuously recorded. The particular geometry chosen for the telescopes makes the two telescopes ABC and A'B'C'D' "cubical" (i.e. the distances AC and A'D' equal the linear dimensions if the square trays), while the threefold telescope A'B'C' is "semi-cubical" (i.e. the distance A'C' is equal to one-half of the tray dimensions). ABCD forms a "narrow-angle telescope", admitting only particles of near-vertical incidence. The dashed lines in the figure indicate the average zenith angle of the particles registered in the three telescopes: obviously the largest for the semi-cubical arrangement. It will be noticed that by recording simultaneously all three rates, we do in fact carry out observations over five distinct average inclinations: By taking the difference between the counting rates of, say, the narrow-angle and the cubical telescope, we record only those particles which pass ABC without traversing D, and hence come with a well-defined zenith angle between the two others.

Now the average energy of the μ-mesons arriving with different inclinations depends in a well-known way on this zenith angle θ; in first approximation it is proportional to $\cos^{-1}\theta$. We can, therefore, ascribe to all our five rates corresponding average energies: the lowest to the narrow-angle telescope, the largest to the semi-

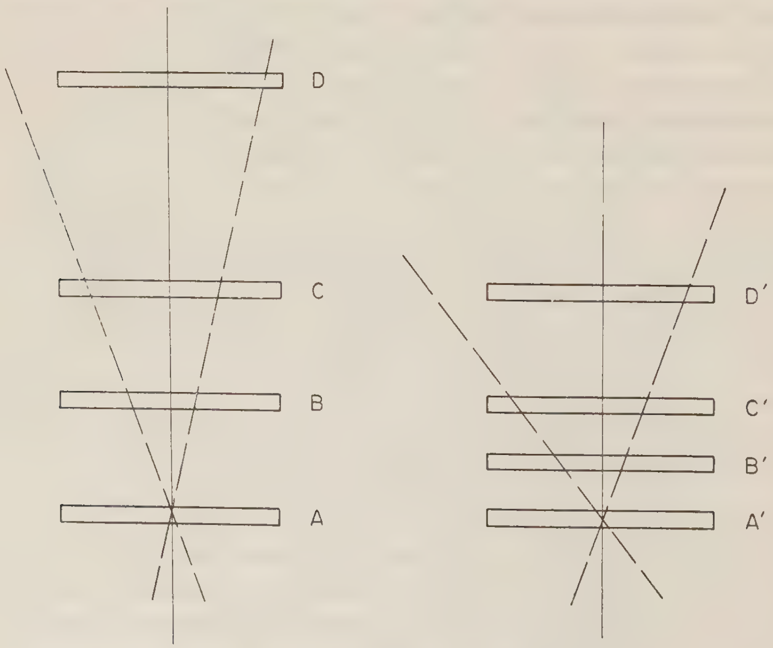

Figure 1
Schematic diagram of the counter arrangement: Narrow-angle telescope ABCD; cubical telescopes ABC and A'B'C'D'; semi-cubical telescope A'B'C'—The dashed lines indicate the average zenith angles of the three arrangements.

cubical setup. But having defined the average meson energy, we can also determine the average energy of the primaries which originated these particles. To use again for simplicity first-approximation expressions, we notice that in the energy region involved ($\sim$50–100 GeV) the number of secondaries n_s in a nuclear collision increases with the primary energy E_0 according to $n_s \propto E_0^{1/4}$ so that we may use the following relation between the average energies E_μ recorded, and the desired primary energies:

$$\bar{E_0} = k\, \bar{E}_\mu^{\,4/3}$$

Consider now the underground telescope. It is of the semi-cubical–cubical variety, defining the same zenith angles as its prototype above ground but demanding penetration of an additonal 30 m.w.e. Writing h_0 for the effective "absorber thickness" of the atmosphere, h for the amount of material traversed in addition to the atmosphere, and θ_i for the average zenith angle of the i-th telescope, we get for the average primary energy $E_0^{(i)}$ of the particles originating the registered μ-mesons

$$E_0^{(i)} = k \left[\frac{h_0 + h}{h_0 \cos \theta_i}\right]^{4/3}$$

h = o for the stations above ground). Figure 2 shows the relative positions, in the

Figure 2

Primary cosmic ray spectrum showing the average primary energies of the mesons in the five groups recorded above ground (open circles), and in the three groups of the underground measurements (full circles).

well-known cosmic ray primary spectrum, of the five average energies defined by the three telescopes above ground, and of the three groups recorded underground. A dashed line indicates the upper limit of the region which can be studied by geomagnetic comparison: it is evident that much new ground is covered in this experiment.

Unfortunately, cosmic ray data can be collected only slowly, and it would be most unwise to jump to conclusions from a still insufficient body of information. Just as an illustration, I may mention that after well over a year of continuous operation, we still do not feel that we have enough material to compute with satisfactory accuracy our own set of coefficients for reduction of the atmospheric effects: our statistics, we feel, is still not good enough. We are, therefore, not yet ready to give final results on other and more interesting phenomena either. However, two examples may be presented which will demonstrate qualitatively the value of the experiment. In Figure 3 and Figure 4 I show the records of the narrow-angle, the cubical, and the semi-cubical telescopes during two periods of some 10–14 days. The first was a period of somewhat higher than normal registered cosmic ray intensity but of no particular solar activity. We are, therefore, not certain whether the observed variation may be purely of atmospheric origin, or may be due to primary fluctuations. The second period represents the "tail end" of a prolonged Forbush decrease, and hence is certainly due to solar activity. In both cases six-hourly averages are plotted in logarithmical scale, so that the *relative* variation can be compared immediately. We note that in Figure 3 no serious and systematic difference is found between the three curves, thus indicating terrestrial origin of the variation. In Figure 4, on the other hand, the differences are rather conspicuous and demonstrate by the smaller effect on the the semi-cubical telescope the decreasing influence of the variation-producing agency on primaries of higher energy. The experiment is being continued, but it will take at least another year before we shall be ready to give detailed results.

A second study, likewise only in its beginning stage, attempts to add information on low-energy primaries to the observations on the high-energy region discussed above. It is based on hitherto unpublished calculations by Segal[5] on the contribution to the ionisation of the upper atmosphere due to cosmic radiation. His results are shown in Figure 5 together with data of some other authors referring to the contribution of other factors. It is seen that in the *D*-layer region, cosmic rays are a very important, perhaps the decisive, contributor. But this effect will, of course, impose no demands on particle energies—other than those following from the geomagnetic cutoff. Its study—the study of variations and fluctuations in the *D*-layer ionization—is therefore an extension to a few GeV of the work already mentioned. However, in order to disentangle the various components, it will also be necessary to observe secular variations on "quiet" periods, and to compare results with those of stations on other geomagnetic location. Some advance in this direction has recently been reported by Bailey[6], but much more remains to be done.

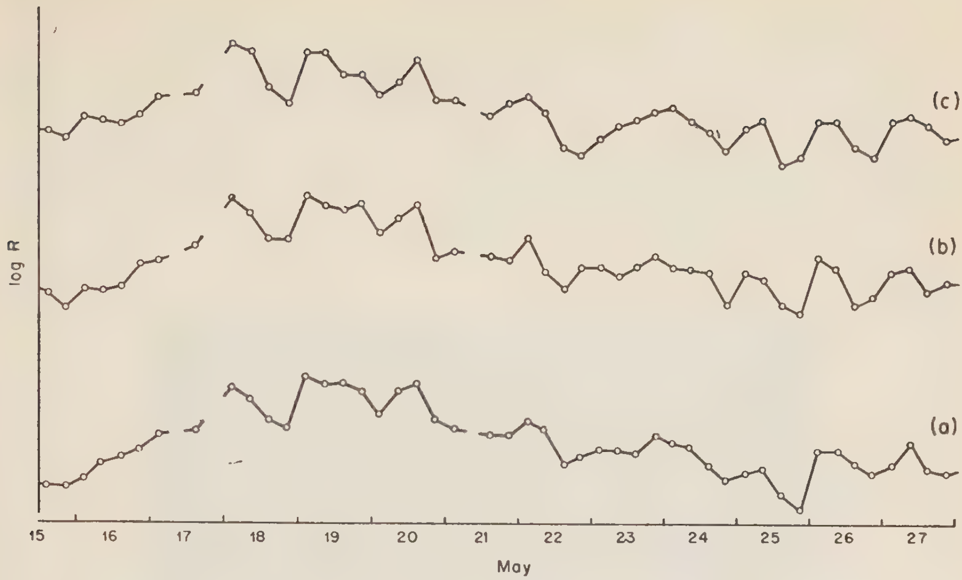

Figure 3
6-hourly rates of the narrow-angle, the cubical, and the semi-cubical telescope during the period May 15–27, 1959, in logarithmical scale.

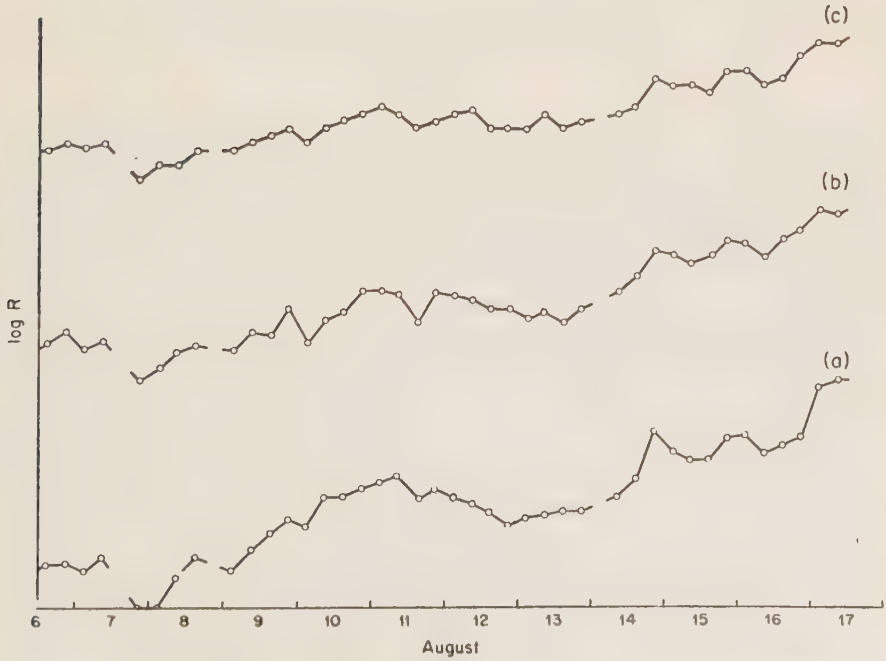

Figure 4
6-hourly rates of the narrow angle, the cubical, and the semi-cubical telescope during the period August 6–18, 1959, in logarithmical scale.

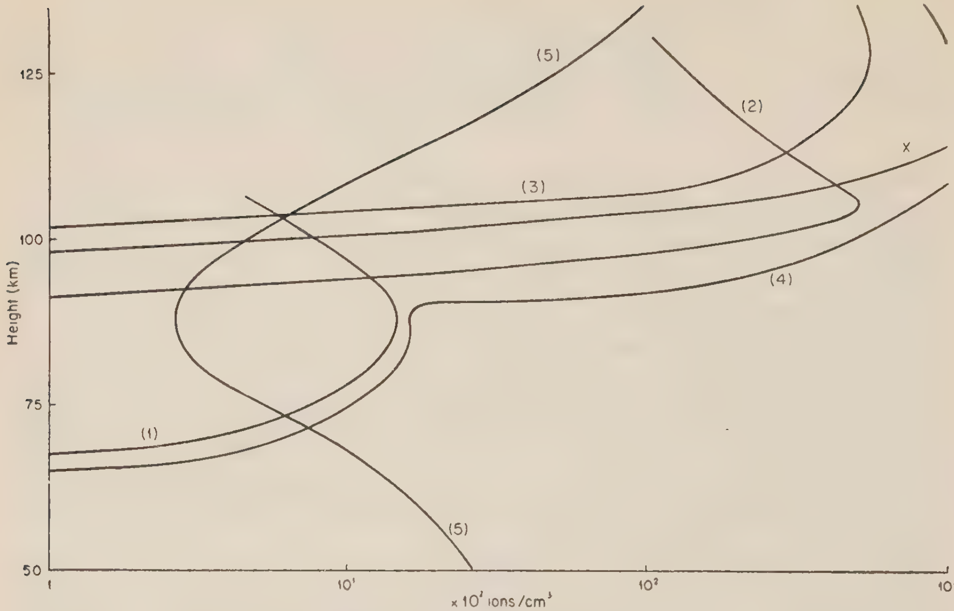

Figure 5
Number of ions per cubic centimeter produced by a number of typical sources; (1) L_{α} — line, (2) L_{β} — line, (3) L — continuum, (4) total electromagnetic solar radiation, x — solar x-rays, (5) cosmic radiation.

The equipment used in our experiment is basically a model C-4 ionospheric recorder, kindly loaned to us by the National Bureau of Standards in Washington. As it is well known, this apparatus emits a powerful pulse of short radio waves, rapidly sweeping through a large frequency region. By timing the echo signal, the distance of layers of a given electron density is then determined. An example of a typical echo sweep is shown in Figure 6.

Unfortunately, we have not been able recently to operate the recorder at full power, since for its new location the antenna has to be built. This is a job which we cannot handle with our own technical staff, and we have not yet succeeded in getting the responsible authorities to act. However, we hope that this will be possible in the very near future, so that we shall be in a position to correlate ionospheric—i.e. low-energy cosmic ray—data with the recordings of meson telescopes.

With the ionospheric we are reaching what might be called the outskirts of the Van Allen belt, and we thus are reminded again of one of the most important outstanding questions: that of the origin of the inner belt. Two radically different views have been offered: according to one (Singer[7]) it results from the decay of albedo neutrons, and thus is relatively stable and time-invariant. According to the other, however (Morrison[8]), the inner belt is not basically different from the outer,

but is like it the result of the capture by the earth of streams of particles emitted by the sun. If this is so, it becomes likely that the inner belt is not constant in time—just as the outer is not—and may even vanish at times, undergoing in general variations and fluctuations in correlation with solar activity.

It is clear from this brief description that simultaneous observations of cosmic ray intensity variations, solar activity, and ionospheric conditions (not only in the

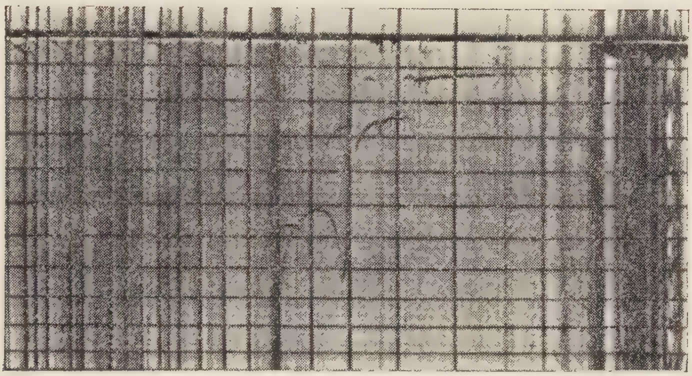

Figure 6
Typical photograph received with the ionospheric recorder, Model C–4.

D-layer, but especially in the highest layers close to the Van Allen belt) are of considerable interest. For this purpose, we plan to put to use an optical telescope equipped with an alpha filter in addition to the two types of apparatus already described. Visual observations will give us information on the state of solar activity, and in some cases a warning before an event, and will be supplemented by the other methods. Unfortunately, here again no systematic results can be reported as yet, since only now the observatory building is at last in construction. Up to this time, we could operate the telescope only by looking out of a laboratory window, and hence for short intervals which made regular observations impossible.

Thus, I am afraid that this report contains a few ideas, a lot of promises, and scarcely any results—none if we ask for definite conclusions. But we must take the data as they come—we cannot manufacture them; and we must work under the conditions we meet — we cannot ignore them. This may or may not be an excuse: but it is an apology, the only one I have to offer for the paucity of this report—and the last one I intend to offer before presenting a fully documented account of our work, perhaps a year from now.

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A PRACTICAL METHOD FOR CALCULATION OF PRESSURE DISTRIBUTION ON AIRFOILS IN SUPERSONIC ROTATIONAL FLOW*

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ABSTRACT

A transformation of coordinates, which introduces Crocco's stream function as independent variable, is used to develop a method for calculation of pressure distribution at the surface of an airfoil in a supersonic rotational flow. The preparation of tables for the practical application of the method is discussed in detail.

NOMENCLATURE

$c^2 = \frac{\gamma - 1}{2}(1 - w^2)$

c_p = pressure coefficient

$g(x)$ = airfoil profile

M_1 = undisturbed flow Mach number

P_p = pressure

R = gas constant

S = entropy

T = temperature

x,y = cartesian coordinates

u,v = velocity components in x and y directions, in units of limiting speed

$w^2 = u^2 + v^2$

γ = ratio of specific heats

δ = flow inclination to undisturbed flow

μ_2 = Mach angle behind wave at airfoil leading edge

ν = shock wave inclination to undisturbed stream

ξ,η = transformed coordinate system

$\Psi(\xi)$ = shock wave profile in (ξ,η)-plane

τ = slope of shock wave

ν_0,δ_0 = value of ν and δ at the leading edge

$(K_{sh}/K_w)_0$ = ratio of shock wave curvature to airfoil curvature ar leading edge.

INDICES

w = condition at airfoil surface; $-$ = condition of undisturbed flow; $_2$ = condition behind shock wave at leading edge

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A method for the evaluation of a second approximation of the pressure coefficient at the surface of an airfoil in a supersonic rotational flow was developed in reference 1. Its main features will be briefly outlined here for ease of reference. The method is based on an application of Crocco's srteam function. The basic equations of flow were transformed by the introduction of the abscissa $\xi = x$ and Crocco's stream function $\eta = \Psi(x,y)$ as independent variables. The ordinate $y = y(\xi,\eta)$ satisfies the following differential equation:

$$u(c^2 - u^2)y_{\xi\xi} - (\gamma - 1)uv(1 - w^2)^{\gamma/(\gamma-1)} y_{\xi\eta} + \frac{\gamma - 1}{2} uw^2(1 - w^2)^{(\gamma+1)/(\gamma-1)} y_{\eta\eta}$$

$$= \frac{1}{2c_p}(w^2 - c^2)(1 - w^2)^{\gamma/(\gamma-1)} \frac{dS}{d\eta} \tag{1}$$

Its derivatives are connected with the velocity components by the equations

$$\begin{cases} y_\xi = \dfrac{v}{u} \\ y_\eta = \dfrac{1}{u(1 - w^2)^{1/(\gamma-1)}} \end{cases} \tag{2}$$

The tangency condition at the airfoil surface is given by

$$y(\xi,0) = g(\xi) \tag{3}$$

where $g(\xi)$ is the known airfoil profile. The boundary conditions at the shock wave surface are expressed by

$$\begin{cases} u = \dfrac{\bar{w}^2 + \dfrac{\gamma - 1}{\gamma + 1}(1 - \bar{w}^2 + \tau^2)}{\bar{w}(1 + \tau^2)} \\ v = \dfrac{\bar{w}^2\tau^2 - \dfrac{\gamma - 1}{\gamma + 1}(1 - \bar{w}^2 + \tau^2)}{\bar{w}\tau(1 + \tau^2)} \end{cases} \tag{4}$$

The values of u, v y_ξ and y_η behind the shock front can be calcuiated by Eqs. (2) and (4) and tabulated as functions of the shock wave slope.

If $\eta = \Psi(\xi)$ represents the shock wave profile in the $(\xi.\eta)$ plane then $d\Psi/d_\xi$ can also be calculated and tabulated as a function of τ:

$$\frac{d\Psi}{d_\xi} = \frac{\tau - y_\xi}{y_\eta} \tag{5}$$

The functions $y(\xi,\eta)$, $u(\xi,\eta)$, $v(\xi\ \eta)$ and $S(\eta)$ are developed in power series of η

$$\begin{cases} y = y_0(\xi) + y_1(\xi)\eta + y_2(\xi)\eta^2 + \dots \\ u = u_0(\xi) + u_1(\xi)\eta + u_2(\xi)\eta^2 + \dots \\ v = v_0(\xi) + v_1(\xi)\eta + v_2(\xi)\eta^2 + \dots \\ S = S_0 + S_1\eta + S_2\eta^2 + \dots \end{cases} \tag{6}$$

Then the pressure coefficient at the airfoil surface is given by

$$c_p = \frac{2}{\gamma M_1^2}\left\{\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}}{(p_2/p_1)^{1/(\gamma-1)}\ (\rho_1/\rho_2)^{\gamma/(\gamma-1)}}\ (1-w_0^2)^{\gamma/(\gamma-1)} - 1\right\} \tag{7}$$

Here p_2/p_1 and ρ_2/ρ_1 are the pressure and density ratios across the shock wave at the leading edge. The value w_0 of the velocity at the airfoil surface is obtained from its value at the shock front by use of expansions (6)

$$\begin{cases} w_0^2 = u_0^2 + v_0^2 \\ u_0 = u - u_1\eta - u_2\eta^2 - \dots \\ v_0 = v - v_1\eta - v_2\eta^2 - \dots \end{cases} \tag{8}$$

Thus in order to obtain a first approximation to c_p we neglect all terms $0(\eta)$ in the above expansions. The velocity components at the airfoil surface are approximated by their values behind the shock wave for the same value of ξ:

$$u_0 = u;\ v_0 = v \tag{9}$$

These are known as functions of the shock wave parameter τ. In order to establish their functional dependence upon ξ it is observed that to this approximation

$$y_\xi(\xi,\eta) \approx y_0'(\xi) = g'(\xi) \tag{10}$$

For a given airfoil geometry and angle of attack $g(\xi)$ is known. The tabulated values of u, v and y_ξ can be used in connection with Eq. (10) in order to obtain $c_p(\xi)$ to a first approximation.

A second approximation to $c_p(\xi)$ is calculated by retaining the first order terms in η in Eqs. (8)

$$u_0 = u - u_1\eta;\ v_0 = v - v_1\eta \tag{11}$$

The first order coefficients u_1 and v_1 are obtained from the expansion of Eqs. (2)

$$\begin{cases} u_1 = \dfrac{u\{2c^2 y_2(\xi) - uvy_\eta y_1'(\xi)\}}{y_\eta(w^2 - c^2)} + 0(\eta) \\[2ex] v_1 = \dfrac{u\{2c^2 y_\xi y_2(\xi) + (u^2 - c^2) y_\eta y_1'(\xi)\}}{y_\eta(w^2 - c^2)} + 0(\eta) \end{cases} \tag{12}$$

Here the coefficient $y_2(\xi)$ is determined from the first approximation to the differential equation (1):

$$\frac{c^2 - u^2}{(1 - w^2)^{\gamma/(\gamma-1)}} y_0''(\xi) - (\gamma - 1) v y_1'(\xi) + (\gamma - 1) w^2 (1 - w^2)^{1/(\gamma-1)} y_2(\xi) = \frac{1}{2\gamma} \frac{w^2 - c^2}{u} \frac{S_1}{c_v} \tag{13}$$

The expressions $y_0''(\xi)$ and $y_1'(\xi)$ appearing in Eqs. (12) and (13) can be obtained by numerical differentiation of the tabulated values of $y_\xi = y_0'(\xi) + 0(\eta)$ and $y_\eta = y_1(\xi) + 0(\eta)$, which are related to ξ by Eq. (10). Similarly η in Eqs. (11) is obtained by numerical integration of Eq. (5), the tabulated values of τ, y_ξ and y_η being related to ξ through Eq. (10).

The parameter S_1/c_v in Eq. (13) is a constant for each undisturbed flow and airfoil geometry and orientation. It is calculated from known flow parameters behind the shock wave at the leading edge:

$$\frac{S_1}{c_v} = \frac{4\gamma(\gamma - 1)}{(\gamma + 1)^2} \frac{[(M_1^2 - 1)\tau^2 - 1]^2 \, g''(0) \, (K_{sh}/K_w)_0}{M_1^2 \, \tau^2 \, (1 + \tau^2) \, [1 + (g'(0))^2]^{3/2} \, w_2 \, (1 - w_2^2)^{\gamma/(\gamma-1)}} \tag{14}$$

$$\left(\frac{K_{sh}}{k_\omega}\right)_0 = \frac{\gamma + 1}{4\cos(\nu_0 - \delta_0)} \frac{1 - \dfrac{\tan^2(\nu_0 - \delta_0)}{\tan^2\mu_2}}{\dfrac{\tan^2(\nu_0 - \delta_0)}{\tan^2\mu_2} + \dfrac{1}{2}\left(\dfrac{\cos^2\mu_2}{\cos^2(\nu_0 - \delta_0)} + \dfrac{1 + \tau^2}{M_1^2 \tau^2}\right)} \tag{15}$$

Finally the dependence of τ and of the other tabulated parameters on ξ is iterated so as to satisfy again the flow tangency condition at the airfoil surface:

$$\frac{v - v_1\eta}{u - u_1\eta} = g'(\xi) \tag{16}$$

In reference 1 a number of numerical examples, covering a wide range of rotational supersonic flows past sharp airfoils, have been worked out numerically and the results were compared with those obtained by the method of characteristics. The present method proved quite reliable and accurate. One of its inherent advantages consists in the fact that most of the numerical work involved in its application could be performed in a general way, independent of the particular airfoil to which it will be applied. Flow parameters like u, v, y_ξ, y_η, etc., behind the shock front can be tabulated once and for all as functions of one common shock wave parameter, e.g. τ. The availability of such tables would greatly simplify the numerical work involved in the application of the method to a particular flow configuration.

The preparation of such numerical tables has been undertaken. They will be calculated by the WEIZAC electronic computer of the Weizman Institute of Science. We shall here discuss some of the details concerning their construction.

Eq. (7) may be written in abbreviated form

$$c_p = \frac{2}{\gamma M_1^2}\left\{\frac{(1-w_0^2)^{\gamma/(\gamma-1)}}{F(\tau_0)} - 1\right\} \tag{17}$$

$$F(\tau) = \frac{\left(\frac{2\gamma}{\gamma+1}M_1^2\frac{\tau^2}{1+\tau^2} - \frac{\gamma-1}{\gamma+1}\right)^{1(/\gamma-1)}\left(\frac{2}{\gamma+1}\frac{1+\tau^2}{M_1^2\tau^2} + \frac{\gamma-1}{\gamma+1}\right)^{\gamma/(\gamma-1)}}{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/(\gamma-1)}} \tag{18}$$

τ_0 being the value of τ at the leading edge.

The expression w_0^2 in Eq. (17) can be developed by substitution from Eqs. (11) to (15) into the following form:

$$w_0^2 = u_0^2 + v_0^2 \tag{19}$$

$$\begin{cases} u_0 = u - (Ay_0'' + BE(\tau_0)g''(0) + Cy_1')\eta \\ v_0 = \{u - (Ay_0'' + BE(\tau_0)g''(0) + \frac{D}{y_\xi}y_1')\eta\}y_\xi \end{cases} \tag{20}$$

with

$$A = \frac{u^3 y_\eta(u^2-c^2)}{w^2(w^2-c^2)}$$

$$B = -\frac{u(1-w^2)}{w^2} \tag{21}$$

$$C = \frac{u^3 y_\xi (w^2 - 2c^2)}{w^2(w^2 - c^2)}$$

$$D = \frac{u[u^2(w^2 - c^2) + c^2 v^2]}{w^2 - c^2}$$

and

$$E = \frac{\gamma - 1}{2(\gamma+1)} \frac{[(M_1^2 - 1)\tau^2 - 1]^2}{M_1^2 (1 + \frac{\gamma-1}{2} M_1^2)\tau^3 (1 + \tau^2)} K, \tag{22}$$

$$K = \frac{1 - \dfrac{\tan^2(\nu - \delta)}{\tan^2\mu}}{w(1 - w^2)^{\gamma/(\gamma-1)}(1 + y_\xi^2)^{3/2} \sin(\nu - \delta)\cos(\nu - \delta) \left[\dfrac{\tan^2(\nu-\delta)}{\tan^2\mu} + \dfrac{1}{2}\left(\dfrac{\cos^2\mu}{\cos^2(\nu-\delta)} + \dfrac{1+\tau^2}{M_1^2\tau^2}\right)\right]}$$

with

$$\begin{cases} \cos^2\mu = \cos^2(\nu - \delta)\{1 - \dfrac{\gamma+1}{2}\tan(\nu - \delta)[\tau - \tan(\nu - \delta)]\}, \\ \tan(\nu - \delta) = \dfrac{\tau - y_\xi}{1 + \tau y_\xi} \end{cases} \tag{23}$$

The following 10 parameters must be calculated and tabulated:

A, B, C, D	(Eqs. 21)
E	(Eqs. 22, 23)
F	(Eq. 18)
u	(Eq. 4)
y_ξ, y_η	(Eq. 2)
$d\Psi/d\xi$	(Eq. 5)

At first sight it would seem natural to choose τ as the independent variable and to calculate these 10 parameters as functions of τ at constant intervals $\Delta\tau$. Such a scheme would not be convenient, however, since it would have complicated the procedure to be followed in the use of the Tables. As seen from Eqs. (20) the calculation of c_p along a given airfoil involves numerical differentiation of $y_0'(\xi)$ and $y_1(\xi)$ and the numerical integration

$$\eta = \int_0^{\xi} \frac{d\Psi}{d\xi} d\xi$$

Should the Tables be computed at constant intervals of τ, these differentiation and integration processes would become quite cumbersome since they would have to proceed numerically at varying intervals of ξ.

Yet there is, of course, no possibility of constructing the tables at constant intervals of ξ since the functional relation $\tau = \tau(\xi)$ is determined by the profile and angle of attack of each individual airfoil.

The practical way out of this difficulty is to calculate the Tables at constant intervals of y_ξ. The differentiations and integration will then be performed with constant increments of y_ξ according to the formulae:

$$\begin{cases} y_0''(\xi) = \dfrac{1}{d\xi/dy_0'} \\ y_1'(\xi) = y_0''(\xi)\dfrac{dy_1}{dy_0'} \\ \eta = \displaystyle\int_{y'_{(0)}}^{y_0'(\xi)} \frac{d\Psi}{d\xi}\frac{1}{y_0''(\xi)}\, dy_0' \end{cases} \tag{24}$$

The application of the tables to the calculation of the second approximation of the pressure coefficient at the airfoil surface will require the following steps:

1. Place origin of (x,y)-plane at airfoil leading edge and draw x-axis in direction of undisturbed flow. Apply Eq. (10) to the given airfoil profile $g(x)$ and calculate $\xi = x$ at constant intervals Δy_ξ of y_ξ.

2. Apply Eqs. (24) to obtain by numerical differentiation and by numerical integration $y_0''(\xi)$, $y_1'(\xi)$ and $\eta(\xi)$, respectively, at constant intervals Δy_ξ of y_ξ.

3. Use the Tables to obtain u, A, B, C and D at constant intervals Δy_ξ.

4. Calculate $g''(0)$; read up from the Tables the value of E at the airfoil leading edge – $E(\tau_0)$.

5. With the values obtained by steps 2, 3 and 4 calculate u_0, v_0 and w_0 at constant intervals Δy_ξ, by Eqs. (19) and (20).

6. Calculate $(1 - w_0^2)^{\gamma/(\gamma-1)}$.

7. Obtain from the Tables the value of F at the airfoil leading edge – $F(\tau_0)$.

8. Calculate by Eq. (17) c_p at constant intervals Δy_ξ.

9. Insert values of u_0 and v_0, as obtained by step (5), into Eq. (16), to obtain the improved correspondence between the calculated values of c_p, step 8, and the abscissae $x = \xi$.

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HIGH-FREQUENCY OSCILLATIONS OF SUPERSONIC AIRFOILS

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ABSTRACT

A study is made of the flow past a supersonic airfoil which oscillates harmonically at high frequency. An asymptotic solution of the linearized equations of motion is obtained by a method analogous to the "eikonal" representation of optical waves. The boundary conditions are satisfied on the surface of the airfoil, so that the effects of airfoil shape are accounted for. The results include (1) the first approximation (for high frequencies) to the velocity potential, (2) the second approximation to the oscillatory pressure on airfoil surface.

EQUATIONS OF MOTION

The supersonic flow past a harmonically oscillating airfoil is governed, within the linear approximation, by the partial differential equation[1]

$$(M^2 - 1)\,\frac{\partial^2\phi^*}{\partial x^2} - \frac{\partial^2\phi^*}{\partial y^2} + 2i\lambda\,\frac{\partial\phi^*}{\partial x} - \lambda^2\phi^* = 0 \tag{1}$$

In eq. (1), as in the following, $\phi^* e^{i\omega t}$ is the oscillatory velocity potential, ω denoting the angular frequency and t the time; x and y are dimensionless Cartesian coordinates, stationary with respect to the airfoil and measured in units of the airfoil chord c, the x axis being parallel to the undisturbed flow; M denotes the free-stream Mach number and λ is the dimensionless "reduced" frequency defined by

$$\lambda = \frac{\omega c}{a_0}$$

where a_0 denotes the speed of sound. The boundary condition which ϕ^* has to satisfy on the airfoil surface can be stated in the general form

$$\left(\frac{\partial\phi^*}{\partial n}\right)_{n=0} = -\,cW \tag{2}$$

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where n denotes the (dimensionless) normal distance from the surface of the airfoil, and $We^{i\omega t}$ is the normal component of the oscillatory velocity of the air on the surface. When the shape and the motion of the airfoil are given, the values of W on its surface are known. In addition to (1) and (2) the velocity potential has to satisfy the "radiation" condition[1] according to which $\phi^* e^{i\omega t}$ must behave at large distances from the airfoil like an outgoing wave of finite energy.

In the following, an asymptotic representation of the velocity potential will be obtained for large values of the reduced frequency λ. For this purpose we put

$$\phi^* = F(x,y;\lambda)\, e^{-i\lambda G(x,y)} \tag{3}$$

and we assume that F does not vary rapidly with position as λ tends to infinity. Thus G will determine primarily the phase, and F the amplitude, of the oscillatory flow; the surfaces $G(x,y) = \text{const}$ will represent the wave fronts. An analogous assumption is made in theoretical optics[2] to account for the transition from wave theory to geometrical optics; the function G is called there the "eikonal". The assumption permits us to expand F in an asymptotic series of powers of λ^{-1}:

$$F = \lambda^{K}[\lambda^{-1}F^{(1)}(x,y) + \lambda^{-2}\, F^{(2)}(x,y) + \ldots]. \tag{4}$$

Substituting (3) and (4) into the differential equation (1) and separating out the resulting powers of λ^{-1} we find that G, $F^{(1)}$, $F^{(2)}$ etc. must satisfy the partial differential equations

$$\left(\frac{\partial G}{\partial y}\right)^2 - (M^2-1)\left(\frac{\partial G}{\partial x}\right)^2 + 2M\frac{\partial G}{\partial x} = 1, \tag{5}$$

$$\frac{\partial G}{\partial y}\,\frac{\partial F^{(1)}}{\partial y} + \left[M - (M^2-1)\frac{\partial G}{\partial x}\right]\frac{\partial F^{(1)}}{\partial x} + \frac{1}{2}\left[\frac{\partial^2 G}{\partial y^2} - (M^2-1)\,\frac{\partial^2 G}{\partial x^2}\right] F^{(1)} = 0, \tag{6}$$

$$\frac{\partial G}{\partial y}\,\frac{\partial F^{(m)}}{\partial y} + \left[M - (M^2-1)\frac{\partial G}{\partial x}\right]\frac{\partial F^{(m)}}{\partial x} + \frac{1}{2}\left[\frac{\partial^2 G}{\partial y^2} - (M^2-1)\,\frac{\partial^2 G}{\partial x^2}\right] F^{(m)} =$$

$$= \frac{1}{2i}\left[\frac{\partial^2 F^{(m-1)}}{\partial y^2} - (M^2-1)\,\frac{\partial^2 F^{(m-1)}}{\partial x^2}\right] \quad (\text{for } m \geqslant 2) \tag{7}$$

Similarly, substituting (3) and (4) into the boundary condition (2) we find that

$$k = 0 \tag{8}$$

and that G, $F^{(1)}$, $F^{(2)}$etc. must satisfy on airfoil surface the conditions

$$(G)_{n=0} = 0, \tag{9}$$

$$\left(\frac{\partial G}{\partial n} F^{(1)}\right)_{n=0} = -icW, \tag{10}$$

$$\left(\frac{\partial G}{\partial n} F^{(m)}\right)_{n=0} = -i\left(\frac{\partial F^{(m-1)}}{\partial n}\right)_{n=0} \text{(for } m \geqslant 2). \tag{11}$$

The equations (5)–(11) show that the required asymptotic expansion of the velocity potential can be obtained by the following procedure. First the phase function $G(x,y)$ is evaluated by solving the equation (5) under the condition (9). When G is known, we evaluate the function $F^{(1)}$ by solving equation (6) under the condition (10), and so on. It should be noted that the "radiation" condition is not taken into account in the determination of the solution; however it will be shown that the resulting solution satisfies the "radiation" condition automatically.

As the boundary conditions (9)–(11) are prescribed on the airfoil surface, the solution will depend on the shape of the airfoil. We represent the surface of the airfoil by the parametric equations

$$x = x_0(\sigma), \quad y = y_0(\sigma) \tag{12}$$

it being assumed that the parameter σ increases along the surface in clockwise direction. We also define the (dimensionless) length $l(\sigma)$ along the surface by

$$l'(\sigma) = [(x_0'(\sigma))^2 + (y_0'(\sigma))^2]^{\frac{1}{2}} \tag{13}$$

where the prime denotes differentiation with respect to σ. The angle $\vartheta(\sigma)$ between the tangent to the surface and the positive x axis will the be given by

$$\sin\vartheta(\sigma) = y_0'(\sigma)/l'(\sigma), \quad \cos\vartheta(\sigma) = x_0'(\sigma)/l'(\sigma) \tag{14}$$

the tangent being assumed to point in the direction of increasing σ. The local (dimensionless) radius of curvature $R(\sigma)$ of the airfoil surface,

$$R(\sigma) = -l'(\sigma)/\vartheta'(\sigma) \tag{15}$$

will also appear in the results.

THE PHASE FUNCTION

We turn now to the evaluation of the phase function $G(x,y)$. Applying to equation (5) the usual method of characteristics for first-order partial differential equations[3] we denote

$$\frac{\partial G}{\partial x} = p, \quad \frac{\partial G}{\partial y} = q, \quad H = \frac{1}{2}[q^2 + 2Mp - (M^2 - 1)p^2 - 1] \tag{16}$$

so that the differential equation (5) becomes

$$H = 0. \tag{17}$$

According to the method of characetristics, the solution of (17) is obtained by solving the system of ordinary differential equations

$$\frac{dx}{ds} = H_p, \quad \frac{dy}{ds} = H_q, \quad \frac{dG}{ds} = pH_p + qH_q, \quad \frac{dp}{ds} = -(pH_G + H_x),$$

$$\frac{dq}{ds} = -(qH_G + H_y) \tag{18}$$

where the subscripts indicate partial differentiation. In our case this system reduces to

$$\frac{dx}{ds} = M - (M^2 - 1)p, \quad \frac{dy}{ds} = q, \quad \frac{dG}{ds} = Mp - (M^2 - 1)p^2 + q^2,$$

$$\frac{dp}{ds} = 0, \qquad \frac{dq}{ds} = 0. \tag{19}$$

The quantity s introduced in (18) or (19) is a parameter which varies along the characteristic lines ("rays") of the solution. The initial conditions for the system (19) are obtained by taking

$$s = 0 \text{ on airfoil surface;} \tag{20}$$

the boundary condition (9) gives then

$$G = 0 \quad \text{when} \quad s = 0 \tag{21}$$

and from (12) we have

$$x = x_0(\sigma), \quad y = y_0(\sigma) \quad \text{when} \quad s = 0. \tag{22}$$

The values $p_0(\sigma)$, $q_0(\sigma)$ which p and q respectively assume on the airfoil surface must satisfy

$$H(p = p_0(\sigma), \ q = q_0(\sigma)) = 0, \tag{23}$$

$$p_0(\sigma)\, x_0'(\sigma) + q_0(\sigma)\, y_0'(\sigma) = 0 \tag{24}$$

(the "strip" relation (24) expresses the fact that G does not vary along the surface). The equations (23) and (24) determine $p_0(\sigma)$ and $q_0(\sigma)$; with the aid of (16) and (14) we find after some calculation that

$$p_0(\sigma) = -\frac{\sin \vartheta(\sigma)}{1 - M \sin \vartheta(\sigma)} \qquad q_0(\sigma) = -\frac{\cos \vartheta(\sigma)}{1 - M \sin \vartheta(\sigma)}. \tag{25}$$

The conditions (21) and (22), together with the statement

$$p = p_0(\sigma), \quad q = q_0(\sigma) \quad \text{when} \quad s = 0 \tag{26}$$

provide a complete set of initial conditions for the differential equations (19). Integrating (19) under these initial conditions we obtain the solution

$$x = x_0(\sigma) + \frac{M - \sin \vartheta(\sigma)}{1 - M \sin \vartheta(\sigma)} s, \quad y = y_0(\sigma) + \frac{\cos \vartheta(\sigma)}{1 - M \sin \vartheta(\sigma)} s \tag{27}$$

$$G = \frac{1}{1 - M \sin \vartheta(\sigma)} s. \tag{28}$$

It should be pointed out here that the airfoil under consideration must have a sharp leading edge and be sufficiently thin to satisfy

$$1 - M \sin \vartheta(\sigma) > 0 \tag{29}$$

along its surface; otherwise a detached shock wave would be formed and the linearized equations of motion would not be valid.

The results (27) and (28) constitute the required solution for the phase function $G(x,y)$. In fact, from eqs. (27) relations of the form $s = s(x,y)$, $\sigma = \sigma(x,y)$ can in principle be obtained, which, when substituted into (28), would give the function $G(x,y)$. Alternately, $G(x,y)$ can be found by eliminating σ from

$$x = x_0(\sigma) + [M - \sin \vartheta(\sigma)]G, \quad y = y_0(\sigma) + \cos \vartheta(\sigma)\, G. \tag{30}$$

The nature of the solution is, however, understood best when (s,σ) are regarded as independent "characteristic" coordinates which define the points (x,y) through the equations (27). When σ is kept constant and s varies from zero to infinity, the equations (27) represent a straight line ("ray") emerging from the point $(x_0(\sigma), y_0(\sigma))$ of the airfoil surface. Because of the restriction (29), the ray lies entirely in the field of flow (so that it does not cross the region occupied by the airfoil). As σ varies, the equations (27) define a system of rays, each ray emerging from a (different) point of the surface. Thus the coordinate σ identifies the various rays while the coordinate s determines the position of a point on its ray. The solution (28) gives then the values of the phase function G on the rays. We notice that G grows linearly from zero to infinity as the distance along a ray increases.

The wave fronts $G(x,y) = \text{const}$ and the direction of the rays can be found by a simple graphical construction (*Figure* 1). We select a value for G, say G_1; then from a point O on the surface we draw the normal OQ with length G_1 and from the end Q of the normal we draw the length MG_1 in the direction of the x-axis, thus

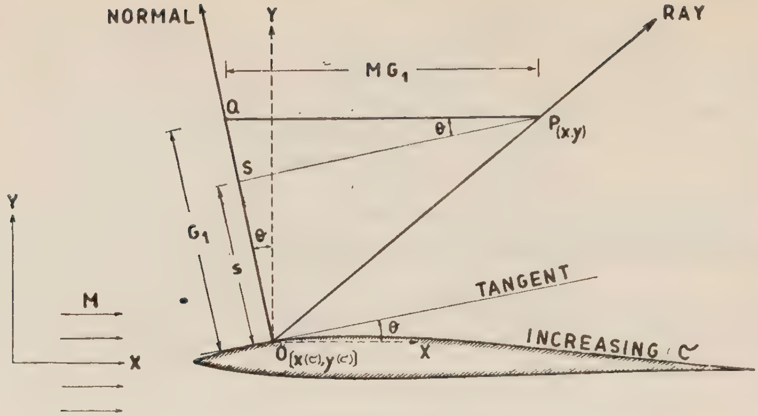

Figure 1

reaching the point P. It follows now from the solution (27), (28) that the point P lies on the ray which emerges from O, and that the value of the function G at the point P is equal to G_1. Furthermore, if we draw PS perpendicular to OQ (*i.e.* parallel to the surface at O) so that the point S lies on the normal OQ, then the length $\overline{OS}$ is equal to the characteristic coordinate s corresponding to the point P. In fact, denoting by (x_0, y_0) and (x,y) the Cartesian Coordinates of O and P respectively, we have from the construction in Figure 1

$$y - y_0 = G_1 \cos \vartheta, \quad x - x_0 = MG_1 - G_1 \sin \vartheta$$

and $\overline{OS} = G_1 - MG_1 \sin \vartheta$, so that $G_1 = \dfrac{1}{1 - M \sin \sigma}\overline{OS}$.

A comparison of these relations with the solution (27), (28) shows at once that the point (x,y) lies on the ray emerging from (x_0, y_0) and that

$$\overline{OS} = s, \quad G(x,y) = G_1.$$

The wave front $G(x,y) = G_1$ can therefore be found by carrying out the construction described above for several points (x_0, y_0) of the surface and by drawing a curve through the resulting points P. It can be proved also that the wave front $G(x,y) = G_1$ is the envelope of circles of radius G_1 with centers at the points $(x_0(\sigma) + MG_1, \; y_0(\sigma))$.

The foregoing solution for the phase function has a simple physical interpretation: it shows that the propagation of disturbances from a surface element of the airfoil is formed by vectorial superposition of two motions, of which one is the propagation with the speed of sound a_0 directed normally to the surface element, while the other is the undisturbed flow moving downstream with the speed Ma_0. Thus the disturbance waves are carried downstream by the main supersonic flow while they propagate away from the airfoil with the speed of sound. It should be noted that the character of the phase function agrees fully with the requirements of the "radiation" condition.

VELOCITY POTENTIAL

Having found the phase function G, we proceed to evaluate the amplitude function $F^{(1)}$ by solving the differential equation (6) under the boundary condition (10). For this purpose it is most convenient to employ the characteristic coordinates (s,σ) as defined in (27) (or in Figure 1). Differentiating the equations and inverting the resulting relations we obtain

$$\frac{\partial\sigma}{\rho x} = \frac{\cos\vartheta}{l'(\sigma)}\left[1 - M\sin\vartheta + \frac{s}{R(\sigma)}\right]^{-1},$$

$$\frac{\partial\sigma}{\partial y} = -\frac{M - \sin\vartheta}{l'(\sigma)}\left[1 - M\sin\vartheta + \frac{s}{R(\sigma)}\right]^{-1}, \tag{31}$$

$$\frac{\partial s}{\partial x} = \left[-\sin\vartheta(1 - M\sin\vartheta) + \frac{M - \sin\vartheta}{1 - M\sin\vartheta}\frac{s}{R(\sigma)}\right.\left[1 - M\sin\vartheta + \frac{s}{R(\sigma)}\right]^{-1},$$

$$\frac{\partial s}{\partial y} = \cos\vartheta\left[1 - M\sin\vartheta - \frac{M^2 - 1}{1 - M\sin\vartheta}\frac{s}{R(\sigma)}\right]\left[1 - M\sin\vartheta + \frac{s}{R(\sigma)}\right]^{-1} \tag{32}$$

where $\vartheta = \vartheta(\sigma)$, $R(\sigma)$ and $l'(\sigma)$ are defined in (13)-(15). The relations (31), (32) allow us to express in terms of s and σ the partial derivatives which appear in the differential equation and the boundary condition for $F^{(1)}$. With the aid of the solution (28) for G, we get

$$\frac{\partial G}{\partial y}\frac{\partial F^{(1)}}{\partial y} + \left[M - (M^2 - 1)\frac{\partial G}{\partial x}\right]\frac{\partial F^{(1)}}{\partial x} = \frac{\partial F^{(1)}}{\partial s}, \tag{33}$$

$$\frac{\partial^2 G}{\partial y^2} - (M^2 - 1)\frac{\partial^2 G}{\partial x^2} = \frac{1}{[1 - M\sin\vartheta(\sigma)]\,R(\sigma) + s} \tag{34}$$

$$\frac{\partial G}{\partial n} = -\frac{\partial G}{\partial x}\sin\vartheta + \frac{\partial G}{\partial y}\cos\vartheta = \frac{1}{1 - M\sin\vartheta(\sigma)}. \tag{35}$$

Therefore the equation (6) and the condition (10) become

$$\frac{\partial F^{(1)}}{\partial s} + \frac{1}{2}\frac{1}{[1 - M\sin\vartheta(\sigma)]\,R(\sigma) + s}F^{(1)} = 0, \tag{36}$$

$$(F^{(1)})_{s=0} = -ic\mathrm{W}(\sigma)[1 - M\sin\vartheta(\sigma)]; \tag{37}$$

in (37) the notation $W(\sigma)$ is introduced to indicate that the normal velocity component W on the surface of the airfoil is a known function of position along the surface. The equation (36) shows that the partial differential equation for $F^{(1)}$ reduces to

an ordinary differential equation along the rays; this is true also with respect to the other functions $F^{(m)}$ since (33) remains valid when $F^{(1)}$ is replaced by $F^{(m)}$. It follows that the motion and the geometry of each surface element of the airfoil determine completely the high-frequency flow on the entire ray which emerges from the element.

Integrating (36) under the boundary condition (37) we obtain the required amplitude function $F^{(1)}$:

$$F^{(1)} = -icW(\sigma)[1 - M\sin\vartheta(\sigma)]\left[1 + \frac{s}{[1 - M\sin\vartheta(\sigma)]\,R(\sigma)}\right]^{-\frac{1}{2}}. \tag{38}$$

This result, together with (4), (8) and (28), gives the first approximation to the velocity potential for high frequencies:

$$\phi^* = -icW(\sigma)\lambda\,[1 - M\sin\vartheta(\sigma)]\left[1 + \frac{s}{[1 - M\sin\vartheta(\sigma)]\,R(\sigma)}\right]^{-\frac{1}{2}} e^{-\frac{i\lambda s}{1 - M\sin\vartheta(\sigma)}} + +0(\lambda^{-2}) \tag{39}$$

The solution (38) shows that the amplitude $F^{(1)}$ of the velocity potential decreases to zero as the distance from the airfoil along a ray tends to infinity, provided that the radius of curvature $R(\sigma)$ is finite at the point from where the ray emerges. When a surface element is flat so that its radius of curvature is infinite, the amplitude $F^{(1)}$ remains constant on the corresponding ray. Therefore, the velocity potential given by (39) represents a wave of finite energy, thus satisfying fully the "radiation" condition.

The higher terms $F^{(m)}$ of the asymptotic expansion (4) can in principle be found in the same way as $F^{(1)}$. Unfortunately the non-homogeneous term in the differential equation (7) turns out to be very complicated, so that higher approximations to the velocity potential in the flow field will not be considered here. We notice, on the other hand, that the boundary condition (11) allows us to find the values of $F^{(2)}$ on the airfoil surface directly from the first approximation $F^{(1)}$given above. This enables us to evaluate the pressure on the surface within second approximation.

PRESSURE ON AIRFOIL SURFACE

We denote by $P^*e^{i\omega t}$ the oscillatory pressure on the surface of the airfoil. The linearized Bernoulli equation for unsteady compressible flow gives

$$P^* = -\frac{\rho_0 a_0}{c}\left(i\lambda\phi^* + M\frac{\partial\phi^*}{\partial x}\right) \tag{40}$$

where ρ_0 is the density of the undisturbed air. Substituting for the velocity potential ϕ^* the high-frequency asymptotic expansion

$$\varphi^* = [\lambda^{-1}F^{(1)} + \lambda^{-2} F^{(2)} + O(\lambda^{-3})]\, e^{-i\lambda G} \tag{41}$$

(eqs. (3), (4) and (8)) and recalling that on the airfoil surface $G = 0$ and $s = 0$, we find that the pressure amplitude P^* on the surface is given by

$$P^* = -\frac{\rho_0 a_0}{c}\left\{i(1 - M\frac{\partial G}{\partial x})F^{(1)} + \lambda^{-1}\left[i(1 - M\frac{\partial G}{\partial x})F^{(2)} + M\frac{\partial F^{(1)}}{\partial x}\right] + 0(\lambda^{-2})\right\}_{s=0} \tag{42}$$

All the terms appearing in this equation can be evaluated by employing the results obtained above. From the solution (28) and the relations (31), (32) it follows that

$$1 - M\frac{\partial G}{\partial x} = \frac{1}{1 - M\sin\vartheta} \tag{43}$$

while the equations (11) (with $m = 2$) and (35) give for $F^{(2)}$ on the surface

$$(F^{(2)})_{s=0} = -\, i(1 - M\sin\vartheta)\left(\frac{\partial F^{(1)}}{\partial n}\right)_{s=0} =$$

$$= -\, i(1 - M\sin\vartheta)\left(\frac{\partial F^{(1)}}{\partial y}\cos\vartheta - \frac{\partial F^{(1)}}{\partial x}\sin\vartheta)\right)_{s=0} \tag{44}$$

When (43) and (44) are substituted into (42), and the partial derivatives of $F^{(1)}$ are expressed (with the aid of (31) and (32)) in terms of the characteristic coordinates we get

$$P^* = -\frac{\rho_0 a_0}{c}\left[i\frac{(F^{(1)})_{s=0}}{1 - M\sin\vartheta} + \lambda^{-1}(1 - M\sin\vartheta)\left(\frac{\partial F^{(1)}}{\partial s}\right)_{s=0} + 0(\lambda^{-2})\right] \tag{45}$$

The values of $F^{(1)}$ and $\partial F^{(1)}/\partial s$ at $s = 0$ are provided by the solution (38). It follows that the oscillatory pressure on the airfoil surface is given, within the second approximation for high frequencies, by the simple result

$$P^* = -\,\rho_0 a_0 W\left[1 + i\frac{1 - M\sin\vartheta}{2R}\lambda^{-1} + 0(\lambda^{-2})\right] \tag{46}$$

where the normal velocity component W, the radius of curvature R and the inclination angle ϑ refer to the same point on the airfoil surface as the pressure P^*. The geometric parameters of the airfoil influence the pressure on its surface mainly through the second term of (46) which may be important when the reduced frequency λ is not

extremely large. It is to be noted that the airfoil shape affects also the distribution of the normal velocity component W on the surface. In the special case of a flat airfoil with zero thickness we have $R = \infty$ so that the formula (46) reduces then to its first term; this term is found to coincide with the high-frequency limit of the Possio result[4] for flat-plate airfoils.

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A HYPERSONIC RAMJET USING A NORMAL DETONATION WAVE

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ABSTRACT

The utility of conventional ramjets at hypersonic speeds is seriously limited due to performance deterioration and growing structural complexity, as flight speed increases.

Some of the associated problems can be alleviated by operating the engine combustion chamber at supersonic mean inlet velocities. The governing equations for engine performance with a normal detonation wave in the combustion chamber are derived, and the flight regime for which combustion through normal detonation is possible is investigated.

Performance of an ideal detonative ramjet using acetylene fuel is presented for flight Mach Numbers of 6 to 10.

A ramjet engine configuration is suggested for performance with normal detonation.

NOMENCLATURE

c_v = exhaust nozzle velocity coefficient

F = engine thrust – lbs

f/a = fuel–air ratio

g = acceleration of gravity

H = total enthalpy

M = Mach number

M_0 = flight Mach Number

M_{D_0} = detonation Mach Number, defined by eq. 14

p = static pressure

P = total pressure

R = gas constant

sfc = specific fuel consumption – lbs/hr/lb

t = static temperature

T = total temperature

v = velocity

v_0 = flight velocity

V_D = detonation velocity

w = engine weight flow – lbs/sec

γ = ratio of specific heats

$\bar{\gamma}$ = average ratio of specific heats

η_d = inlet efficiency

ρ = density

Stations

0 = free stream stagnation

2 = entry to detonation wave

3 = exit from detonation wave

4 = nozzle exit

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INTRODUCTION

Considerable attention has been given recently to the use of air-breathing engines in missions normally reserved for rocket-powered vehicles. Aside from subsonic and supersonic cruising missions, in which the air-breathing engine's superiority is unchallenged, the possibility of using them for booster or hypersonic cruise missions merits further investigation because of their low fuel consumption. Both of these missions require engine operation at hypersonic speeds, a region as yet unexplored by air-breathing engines.

As is well known current material limitations restrict the maximum allowable turbine entry temperature for turbojet engines to about 2000° F. Due to this limitation the turbojet cycle is less efficient than the ramjet cycle at speeds above Mach 4[1]. Therefore the appropriate air-breathing powerplant for hypersonic speeds is the ramjet, which on occasion has been characterized as a "flying stove-pipe". However, it is questionable whether the simplicity and low weight generally associated with ramjet engines can be maintained by conventional ramjets designed for hypersonic speeds. The following are some of the principal difficulties encountered:

a) *Inlet*

For subsonic combustor performance, the ratio of inlet capture area to combustor area rises sharply with flight Mach Number. The capture area ratio for a perfect inlet is shown in Figure 1. For attainable inlet efficiencies this ratio would, of course, be lower; but, as can be readily seen, the inlet geometry requirements become very severe at high speeds. If reasonably good inlet performance is required at lower speeds also, say during vehicle acceleration, a variable geometry inlet becomes a necessity. Due to the very high air temperatures encountered at hypersonic speeds, some of the inlet surfaces will require cooling. In light of these considerations, it can be seen that the inlet will constitute a heavy and complex engine component.

b) *Exhaust nozzle*

At hypersonic speeds the exhaust nozzle operates at pressure ratios above 100. Exit to throat area ratios required for full expansion are shown in Figure 2. The severe geometry requirements are compounded by the high gas temperatures in the nozzle. As for the inlet, good performance over a range of flight speeds requires considerable variation in nozzle geometry.

c) *Combustion chamber*

Since combustor inlet temperatures exceed 1500° F, the chamber requires regenerative cooling. In addition, the high chamber temperatures place a limit on the possibility of converting chemical fuel energy into thermal energy of the gas stream. This limit is set by dissociation and ionization reactions in the gas stream at high temperatures. As a result, progressively less of the fuel energy is available for raising the thermal

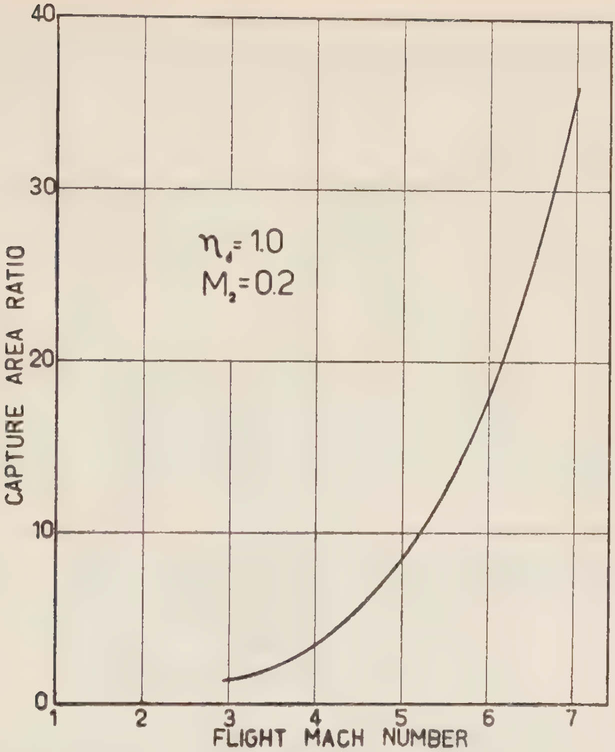

Figure 1
Inlet capture area to combustion chamber area ratio.

energy of the gas stream. Unless the energy expended on dissociation and ionization can be reconverted to thermal energy by reassociation reactions as the gas cools in the exhaust nozzle, it makes no contribution to the propulsive thrust of the engine. Although the degree of reassociation to be expected in the combustion products of fuel-air mixtures is not completely determinate at this time, there seems little doubt that high-temperature dissociation and ionization places an upper limit on the speed at which useful performance can be obtained from conventional ramjets.

THE DETONATION RAMJET

The difficulties associated with conventional ramjet operation at hypersonic speeds can be alleviated to some extent if the combustion process is carried out at supersonic gas velocity. Little or no diffusion would then be required prior to combustion, the exhaust nozzle would not require a convergent section, and gas temperatures throughout the engine would be considerably lower.

Conventional ramjets have been restricted to low subsonic combustor inlet velocities, due to the very low velocities of flame propagation associated with the deflagration of fuel–air mixtures. However, combustion waves moving with supersonic velocities

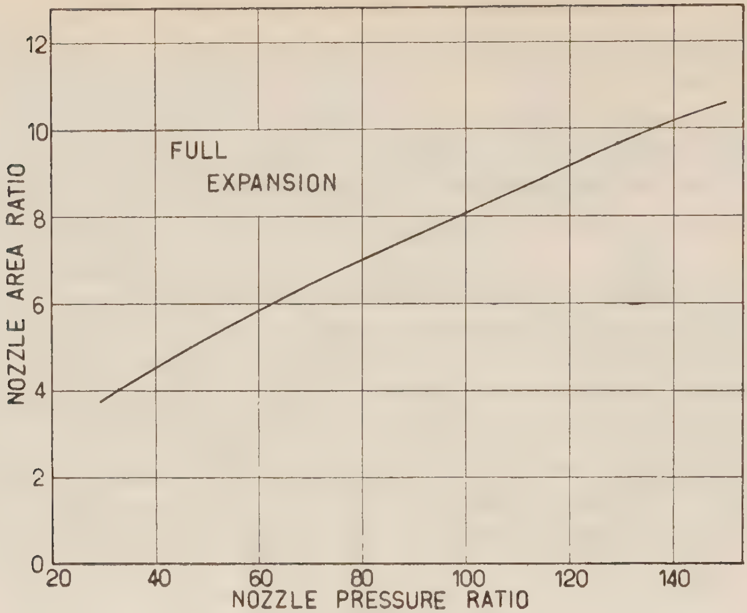

Figure 2
Nozzle exit to throat area ratio

(detonations) were first observed over fifty years ago, and have been intensively studied since. These studies were restricted to detonation waves propagating into stationary gases. Recently, however, the possibility of stabilizing detonation waves in supersonic gas streams has been investigated, and the application of standing detonation waves to ramjet propulsion has been suggested[2, 3]. This has been followed by experimental work in which detonation waves have been successfully stabilized in supersonic gas streams[4, 5].

The standing detonation wave is analogous to a standing shock wave established in a supersonic stream, and it has been suggested that it is actually a shock wave followed by a subsonic combustion wave[6]. Although on the basis of aero-thermodynamic considerations alone an infinite number of supersonic detonation velocities are possible for a given combustible mixture of gases, only waves travelling at sonic velocity with respect to the burned gas, Chapman-Jouget detonations, have been observed in experiments on initially stationary mixtures[7].

In a manner analogous to shock wave stabilization in supersonic diffusers, either normal or oblique detonation waves may be stabilized in the combustion chamber. For a particular detonable mixture, the total pressure loss (or entropy gain) of the combustion process will be minimized for high detonation wave angles with respect to the mean flow direction, since they occur at the lowest Mach Number consistent with Chapman-Jouget detonation. The optimum wave angle for best engine efficiency will, however, depend on the inlet efficiency. An estimate of the upper limit

of detonative ramjet performance can be obtained by assuming normal Chapman-Jouget detonation in the combustion chamber and perfect component performance.

EQUATIONS FOR NORMAL DETONATION RAMJET

Considering the station designations given in Figure 3, the engine exhaust velocity v_4, is given by

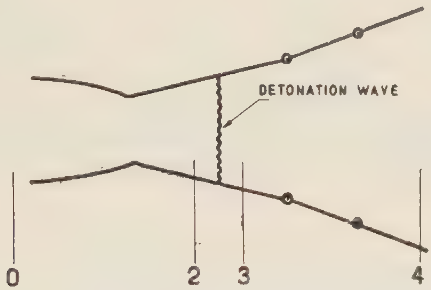

Figure 3
Station Designation for detonative ramjet

$$v_4 = c_v \sqrt{2H_0 \frac{H_3}{H_0}\left[1 - \left(\frac{1}{\eta_d}\frac{p_0}{P_0}\right)^{\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_{34}}} \left(\frac{P_2}{P_3}\right)^{\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_{34}}}\right]} \tag{1}$$

where $\bar{\gamma}_{34}$ signifies the average specific heat ratio between stations 3 and 4.

Assuming one-dimensional flow, perfect gas behaviour, and neglecting changes in molecular weight, the total enthalpy ratio across the detonation wave is obtained from the first law of thermodynamics as

$$\frac{H_3}{H_2} = \frac{\gamma_3}{\gamma_2}\frac{(\gamma_2-1)}{(\gamma_3-1)}\frac{t_3}{t_2}\frac{\left(1 + \frac{\gamma_3-1}{2}M_3^2\right)}{\left(1 + \frac{\gamma_2-1}{2}M_2^2\right)} \tag{2}$$

From the one-dimensional momentum equation connecting stations 2 and 3.

$$\frac{t_3}{t_2} = \frac{\rho_2}{\rho_3}\frac{1+\gamma_2 M_2^2}{1+\gamma_3 M_3^2} \tag{3}$$

which by use of the continuity equation across the wave may be expressed as

$$\frac{t_3}{t_2} = \frac{M_3^2}{M_2^2}\frac{\gamma_3}{\gamma_2}\frac{(1+\gamma_2 M_2^2)^2}{(1+\gamma_3 M_3^2)^2} \tag{4}$$

Substituting equation (4) into equation (2) gives

$$\frac{H_3}{H_2} = \left(\frac{\gamma_3}{\gamma_2}\right)^2 \frac{(\gamma_2 - 1)}{(\gamma_3 - 1)} \frac{(1 + \gamma_2 M_2^2)^2}{(1 + \gamma_3 M_3^2)^2} \frac{(1 + \frac{\gamma_3 - 1}{2} M_3^2)}{(1 + \frac{\gamma_2 - 1}{2} M_2^2)} \frac{M_3^2}{M_2^2} \tag{5}$$

But for a Chapman-Jouget detonation, the Mach Number of the burned gases, M_3, is one, and equation (5) therefore, takes the form

$$\frac{H_3}{H_2} = \frac{1}{2M_2^2} \left(\frac{\gamma_3}{\gamma_2}\right)^2 \left(\frac{\gamma_2 - 1}{\gamma_3^2 - 1}\right) \frac{(1 + \gamma_2 M_2^2)^2}{(1 + \frac{\gamma_2 - 1}{2} M_2^2)} \tag{6}$$

Considering the flow to be adiabatic between the inlet and the combustion chamber, equation (6) expresses the total enthalpy rate between station 3 and 0, i.e. H_3/H_0.

The total pressure ratio across the detonation wave is given by

$$\frac{P_3}{P_2} = \frac{p_3}{p_2} \frac{\left(1 + \frac{\gamma_3 - 1}{2} M_3^2\right)^{\frac{\bar{\gamma}_3}{\bar{\gamma}_3 - 1}}}{\left(1 + \frac{\gamma_2 - 1}{2} M_2^2\right)^{\frac{\bar{\gamma}_2}{\bar{\gamma}_2 - 1}}} \tag{7}$$

where the average ratio of specific heats, $\bar{\gamma}$, is to be evaluated between the static and stagnation temperatures at the station in question.

From the one-dimensional momentum equation across the wave

$$\frac{p_3}{p_2} = \frac{1 + \gamma_2 M^2}{1 + \gamma_3 M_3^2} \tag{8}$$

and substituting equation (8) into equation (7), we obtain the total pressure ratio across the detonation wave as

$$\frac{P_3}{P_2} = \frac{1 + \gamma_2 M_2^2}{1 + \gamma_3 M_3^2} \frac{\left(1 + \frac{\gamma_3 - 1}{2} M_3^2\right)^{\frac{\bar{\gamma}_3}{\bar{\gamma}_3 - 1}}}{\left(1 + \frac{\gamma_2 - 1}{2} M_2^2\right)^{\frac{\bar{\gamma}_2}{\bar{\gamma}_2 - 1}}} \tag{9}$$

For a Chapman-Jouget wave this takes the form

$$\frac{P_3}{P_2} = \frac{1 + \gamma_2 M_2^2}{\gamma_3 + 1} \frac{\left(\frac{\gamma_3 + 1}{2}\right)^{\frac{\bar{\gamma}_3}{\bar{\gamma}_3 - 1}}}{\left(1 + \frac{\gamma_2 - 1}{2} M_2^2\right)^{\frac{\bar{\gamma}_2}{\bar{\gamma}_2 - 1}}} \tag{10}$$

If viscous and mixing losses are neglected, equation (10) gives the total pressure ratio for the engine combustion chamber.

Substituting equation (6) and (10) into equation (1) gives the engine exhaust velocity as

$$v_4 = c_v \left(\frac{\gamma_3}{\gamma_2}\right) \frac{(1+\gamma_2 M_2^2)}{M_2} \sqrt{\frac{(\gamma_2-1)}{(\gamma_3^2-1)} \frac{H_0}{1+\frac{\gamma_2-1}{2}M_2^2}}$$

$$\sqrt{1-\left(\frac{1}{\eta_d}\frac{p_0}{P_0}\right)^{\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_{34}}} \left[\frac{1+\frac{\gamma_2-1}{2}M_2^2}{\frac{\gamma_3+1}{2}}\right]^{\frac{\bar{\gamma}_2}{\bar{\gamma}_{34}}\cdot\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_2-1}} \left(\frac{\gamma_3+1}{1+\gamma_2 M_2^2}\right)^{\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_{34}}}} \tag{11}$$

In order that the wave be stabilized, the gas velocity entering the detonation wave must equal the detonation velocity of the fuel-air mixture,

$$M_2 = \frac{V_d}{\sqrt{\gamma_2 R t_2}} \tag{12}$$

or

$$M_2 = M_{D_0} \sqrt{\frac{\gamma_0-1}{\gamma_2-1}\left(1+\frac{\gamma_2-1}{2}M_2^2\right)} \tag{13}$$

where the detonation Mach Number, M_{D_0}, is defined as

$$M_{D_0} = \frac{V_D}{\sqrt{\gamma_0 R T_0}} \tag{14}$$

Therefore the combustion chamber Mach Number for stabilization of a normal Chapman-Jouget detonation wave is given by

$$M_2^2 = \frac{\frac{\gamma_0-1}{\gamma_2-1}M_{D_0}^2}{1-\frac{\gamma_0-1}{2}M_{D_0}^2} \tag{15}$$

and substituting this equation into equation (11) gives the following expression for the engine exhaust velocity

$$v_4 = c_v \left(\frac{\gamma_3}{\gamma_2}\right) \frac{(\gamma_2-1)}{M_{D_0}} \left[1+\frac{(\gamma_0-1)(\gamma_2+1)}{2(\gamma_2-1)} M_{D_0}^2\right] \sqrt{\frac{H_0}{(\gamma_0-1)(\gamma_3^2-1)}} \tag{16}$$

$$\sqrt{1-\left(\frac{1}{\eta_d}\frac{p_0}{P_0}\right)^{\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_{34}}} \frac{(2)^{\frac{\bar{\gamma}_3}{\bar{\gamma}_{34}}\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_3-1}}}{\left[1+\frac{(\gamma_0-1)(\gamma_2+1)}{2(\gamma_2-1)}M_{D_0}^2\right]^{\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_{34}}} \left[1-\frac{\gamma_0-1}{2}M_{D_0}^2\right]^{\frac{1}{\bar{\gamma}_{34}}\frac{\bar{\gamma}_{34}-1}{\bar{\gamma}_3-1}} (\gamma_3+1)^{\frac{1}{\bar{\gamma}_{34}} \cdots}}}$$

The specific thrust and specfic fuel consumption can be obtained from the exhaust velocity by the well known relations

$$\frac{F}{w} = \frac{(1+f/a)\,v_4 - v_0}{g} \tag{17}$$

$$sfc = \frac{3600\,f/a}{F/w} \tag{18}$$

DETONATIVE RAMJET PERFORMANCE WITH ACETYLENE FUEL

The ideal performance of a detonative ramjet using acetylene fuel is shown in Figure 4. In calculating the performance perfect component efficiency was assumed. The thermodynamic properties of the fuel–air mixture were assumed identical with those of air in chemical equilibrium as given in reference 8. The detonation velocities of acetylene–air mixtures were taken from reference 7.

Figure 5 is a plot of M_0/sfc, a measure of overall engine efficiency, using the minimum *sfc* for each Mach Number as given in Figure 4. Since the heating value of acetylene is of the same order as conventional jet engine fuels, the performance shown in Figure 5 may be compared to the known performance of conventional ramjets at supersonic speeds. In making this comparison it should be recalled that the operating inlet efficiency of the detonative ramjet should be considerably superior to that of the conventional ramjet due to the much smaller amount of diffusion required.

It can be seen that the engine efficiency of the detonative ramjet at hypersonic speeds exceeds that of conventional ramjets at supersonic speeds. As far as fuel consumption is concerned, it may therefore be anticipated that hypersonic cruising using a detonation ramjet will be more economical than cruising at supersonic or even subsonic speeds using conventional powerplants.

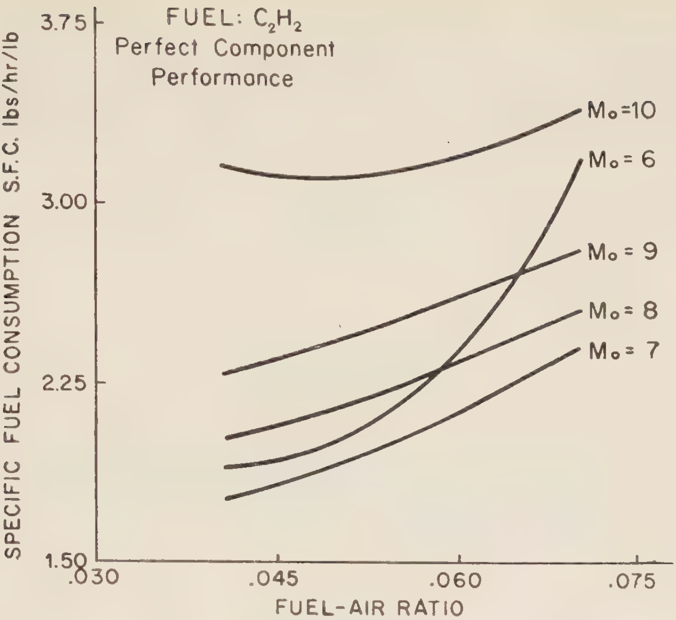

Figure 4
Specific fuel consumption of ideal detonative ramjet using acetylene

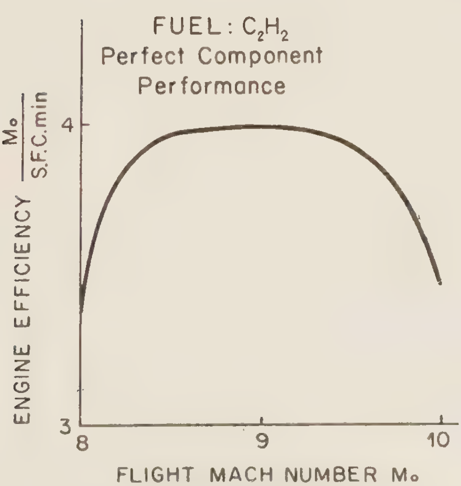

Figure 5
Optimum engine efficiency of ideal detonative ramjet using acetylene

The detonative ramjet, like the conventional ramjet, is inoperative below a minimum flight speed limit. This limit is the minimum speed at which stable detonation can be established for the leanest detonable fuel–air mixture. In other words, the detonation velocity must not exceed the maximum flow velocity attainable in the engine. Ideally this limit is therefore given by

$$\sqrt{\frac{2\gamma_0}{\gamma_0 - 1} R T_0} = V_D \tag{19}$$

or

$$\sqrt{\frac{2}{\gamma_0 - 1}} = M_{D_0} \tag{20}$$

The maximum permissible detonation velocity as a function of flight Mach Number at 100,000 feet is shown in Figure 6. The detonation velocities of two acetylene–air mixtures are also shown in the figure.

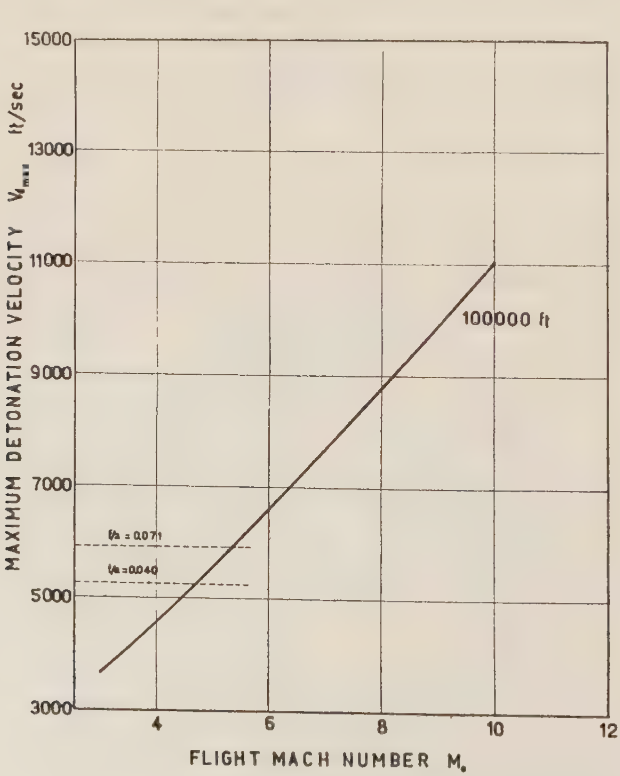

Figure 6
Maximum allowable detonation velocity for standing detonation wave

ENGINE CONFIGURATION

A schematic layout of a ramjet using a normal detonation wave is shown in Figure 7. The area variation of the combustion chamber must be sufficiently large so as to allow stabilization of the detonation wave at minimum flight speed and maximum fuel–air ratio, as well as at maximum flight speed and minimum fuel–air ratio. It is to be expected that the detonation wave, much like a normal shock in a supersonic diffuser, will seek its own position of equilibrium. Increasing fuel–air ratio at fixed

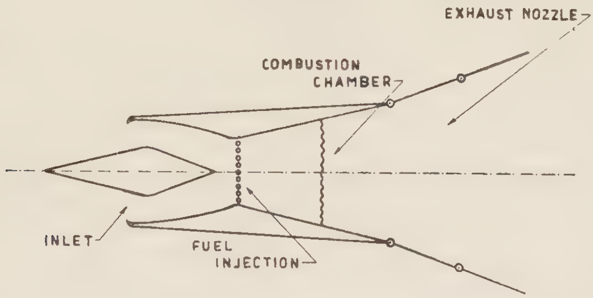

Figure 7
Detonative ramjet

flight speed will force the wave towards the exhaust nozzle, whereas increasing flight speed at fixed fuel–air ratio will tend to suck the wave in the direction of the inlet.

Should the engine be required to operate at flight speeds sufficiently low so that stable detonations are either not feasible or detonative performance is poor, it is possible to design the engine to be capable of alternating between detonative combustion and conventional subsonic combustion. By forming a throat in the exhaust nozzle a normal shock will be forced into the inlet and subsonic flow established in the combustion chamber, allowing conventional operation.

CONCLUSIONS

Some of the difficulties to be encountered by conventional ramjets at hypersonic speeds can be alleviated by designing the engine for supersonic mean entry velocities into the combustion chamber utilizing a stabilized detonation wave. The efficiency of such an engine is competitive with that of conventional ramjets at supersonic speeds. It is possible to size the engine geometry so as to permit alternating between the conventional and detonative mode of operation.

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SOME DEVELOPMENTS OF THE END-BURNING CHARGE

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INTRODUCTION

The magnitude of thrust obtainable in the usual type of the end-burning charge is limited, once the type of propellant and grain diameter is given. An attempt to increase the thrust of an end-burning charge rocket motor by means of increasing the burning surface will fail because of the strongly regressive characteristic of the first phase of any such modified surface. The two possible burning surface modifications, of the concave or convex type recede in general into a surface equal approximately to the circular cross section of the grain.

The purpose of this paper is to indicate some possilbilities of a stable thrust augmentation of an end-burning grain by means of a special grain design.

END BURNING GRAIN CHARACTERISTICS

The usual end-burning grain consists of a propellant cylinder inhibited against burning on one of his bases and on the lateral surface exposing the other uninhibited base to burning. The burning front is generated as an envelope of spheres whose radii are the instantaneous burning distances (Figure 1). Applying this rule of burning front formation to an increased initial surface of an end-burning grain

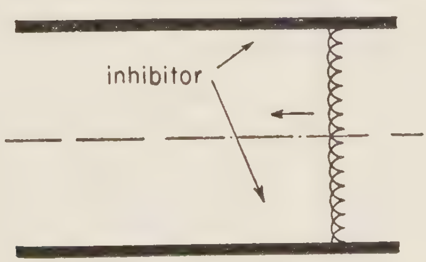

Figure 1

we reach the following conclusion:

a) A convex surface, generated by the revolution of a curve around the axis of the grain transforms itself in the course of burning in a concave-convex flat surface with a cup; this surface recedes ultimately into a circular surface equal to the cross-section of the grain (Figure 2a).

b) A concave surface, generated by the revolution of a curve around the axis of the grain with angles of intersection with the axis of the grain, transforms itself

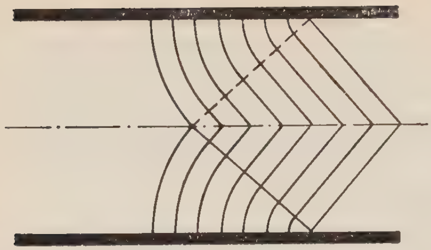

Figure 2a

Figure 2b

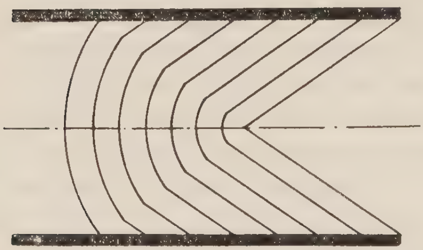

Figure 3a

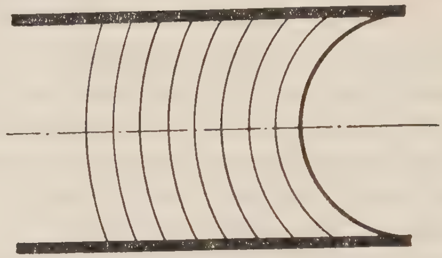

Figure 3b

into a concave spheroidal surface and ultimately into the cross-section of the grain (Figure 3a).

The transformation of the convex surfaces during burning is due to the fact that the burning front at the point of contact with the inhibitor (restriction) generates a toroidal surface which expands until it reaches the axis of the grain. This surface is predominant and eliminates any previous forms (Figure 2).

The transformation of the concave surfaces is due to the fact that any angle (e.g. the angle of the generating curve with the axis) serves as a focus generating toroidal or spherical surfaces which dominantly transform the surface into a spherical concave surface approaching ultimately the cross-section of the grain (Figure 3b).

STABILISATION OF THE BURNING SURFACE

We shall consider a positive (convex) cone surface burning towards the base of the grain (Figure 4a). The points of contact of the surface will generate an enveloped toroidal surface unless prevented by inserting between the conical surface and the restrictive layer a cylindrical layer of a propellant having a higher burning rate than the main grains. In such a case the points of contact between the two propellants will not become foci of generation of the receding toroidal surface because the faster burning layer will reveal the surface of the slower burning propellant parallel to the inhibitor, before the main burning front reaches the point of contact. Figure 4b shows the formation of the new burning fronts at intervals t: v_1, v_2 indicating the two burning rates. In the same drawing we can see that a new cone is generating, having a different half angle than the original one. The angle of the new cone is obviously

$$\mathrm{tg} = \frac{v_1}{v_2} = \mu$$

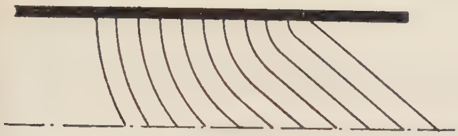

Figure 4a

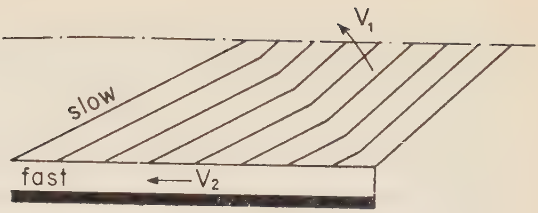

Figure 4b

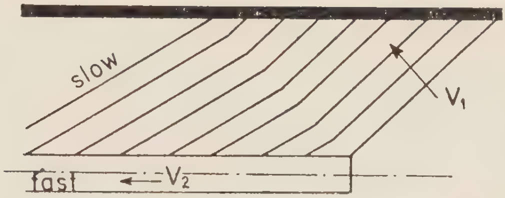

Figure 4c

Choosing the half angle of the slow burning propellant cone equal to the ratio of the burning rates of the two propellants, we shall obtain a constant burning surface. The ratio between the circular grain cross-section, usually serving as the burning surface and the increased conical surface (neglecting the contribution of the thin fast burning layer) is given for a positive cone by:

$$\varphi = \frac{\sqrt{1 - \left(\frac{v_1}{v_2}\right)^2}}{\frac{v_1}{v_2}}$$

The enclosed graph (Figure 5) shows the dependance of the ratio between circular grain section area and the conical developed area and the burning rates ratio. It

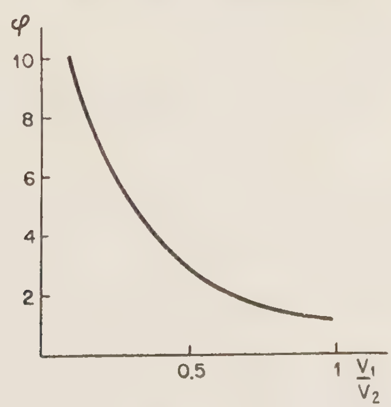

Figure 5

appears obvious that an inverse disposition of the fast and slow burning propellant layers will behave as a normal convex cone end burning charge, the slow burning propellant acting as a burning inhibitor. A negative cone with an inserted central fast burning cylindrical core will behave in a similar manner stabilizing itself at a surface of a truncated cone. Keeping the ratio of the inner fast burning core diameter to the outer diameter small, we can consider the generated surface as being one of a cone (Figure 4c).

FURTHER APPLICATIONS OF THE METHOD

The mounting of the described conical charges in a rocket motor case might present difficulties in design or produce a prohibitive tail-off at the end of the burning.

It seems, therefore, desirable to shorten the reaction zone. This can be achieved using a cylindrical layer of a fast burning propellant embedded in the slower burning charge (Figure 6a). A further shortening of the reaction zone could be obtained using a combination of a central core and several concentric cylinders made of a fast burning propellant and embedded in the slow burning grain (Figure 6a, 6b).

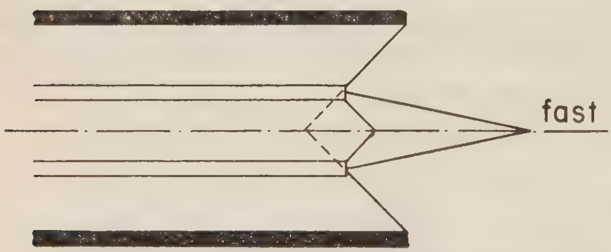

Figure 6a

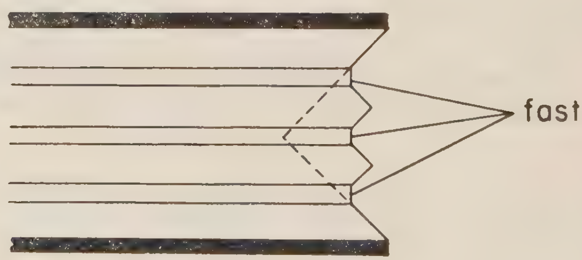

Figure 6b

The extension of this procedure cannot, of course, proceed ad infinitum, the thickness of the interposed slow burning layers being the limitation. The amount of layers depends on the capacity of the propellant to burn in thin layers without breaking up as well as on the technological difficulties arising in the manufacture of the grains.

Another employment of the layer technique consists in the possibility of a limited thrust programming. To achieve this scope it would be necessary to use layers and cores of fast burning propellant of variable thickness.

The calculation of the generated surface bases itself on two assumptions:

a) the burning front generates along the instantaneous radii of curvature; the rate of change of the radius of curvature with time being the rate of burning (slow propellant) and assumed constant.

b) the burning front of the fast propellant is assumed always perpendicular to the x-axis and assumed constant as well.

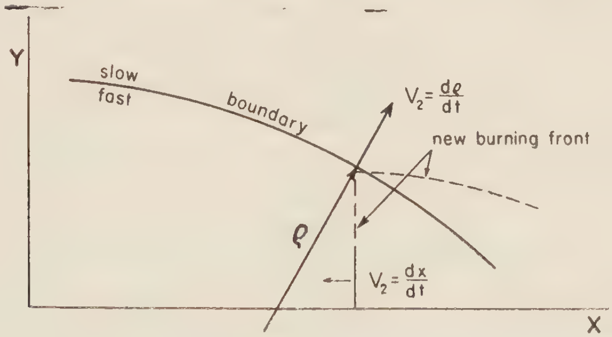

Figure 7

Then, setting the following nomenclature:

$y = f(x)$, the equation of the curve of contact between the fast and slow propellants, the fast propellant assumed located between the curve and the x-axis, and

y', y'', y''', first second and third derivatives of the $y = f(x)$ curve with respect to x,

$\rho = \dfrac{(1+y'^2)^{3/2}}{y''}$, the radius of curvature of the said curve.

$\dfrac{d\rho}{dt} = v_1$, the rate of burning of the slow propellant taken with the appropriate sign according to the character of the curve,

$\dfrac{dx}{dt} = v_2$, the rate of burning of the fast propellant assumed constant as well, and finally –

$\dfrac{v_1}{v_2} = \mu$, the ratio of the two burning rates.

we obtain, after derivation of the radius of curvature with respect to x:

$$\frac{d\rho}{dx} = \frac{3(1+y'^2)^{\frac{1}{2}}\; y'y''^2 - (1+y'^2)^{3/2}y'''}{y''^2} \tag{1}$$

and introducing the two propellant burning rates:

$$\frac{d\rho}{dt}\cdot\frac{dt}{dx} = \mu = \frac{3(1+y'^2)^{\frac{1}{2}}y'y''^2 - (1+y'^2)^{3/2}y'''}{y''^2} \tag{1a}$$

Solving at first with respect to y'' and y' we have after separation of variables:

$$\frac{dy''}{y''} = \frac{3}{2} \frac{dy'^2}{1 + y'^2} - \frac{dy'}{(1 + y'^2)^{3/2}} C_1 \tag{2}$$

Integrating, we obtain:

$$\ln \frac{y''}{C_1(1+y'^2)^{3/2}} = \frac{-y'}{(1+y'^2)^{\frac{1}{2}}} \tag{3}$$

Separating again the variables, we have:

$$dy' \exp \frac{\dfrac{y'}{(1 + y'^2)^{\frac{1}{2}}}}{C_1 (1 + y'^2)^{3/2}} = x + C_2 \tag{4}$$

The expression (4) gives the generated curve in items of its slope and x.

The final integration giving the explicit form of y as function of x is obtained by quadratures. The area is obtained later by a simple quadrature of

$$A = 2\pi \int_{x1}^{x2} yds$$

(x_1, x_2 being the two reference coordinates of the curve involved).

It has to be stressed that the final integration of expression (4) has to be done by development into series of the integral curve which yields:

$$y' = \frac{\ln [2C_1 (x + C_2)]^{1/\mu}}{\{1 - [\frac{1}{\mu}\ln 2C_1 (x + C_2)]^2\}^{\frac{1}{2}}} \tag{4a}$$

The exact analytical treatment of the solution might present some difficulties especially if the basic curve of contact between the two propellants will present discontinuities or singular points. In such a case, as well as in cases where the precision of the solution is of limited importance, it appears advisable to recur to the graphical solution using calculated radii of curvature as reference points.

CONCLUSIONS

The described method of end-burning charge surface augmentation can offer a vast game of applications in sustainer rocket motors. The fabrication difficulties of concentrical charges vary with the type of propellant and in case of cast type composite propellants should not present excessive complications.

IONIZATION MEASUREMENTS IN SHOCKED GASES

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The study of electrical properties of thermally ionized gases produced by shock waves, was initiated by A. R. Kantrowitz and his collaborators[1–6], first at Cornell, later at AVCO.

The problem was thoroughly investigated for the first time by S. C. Lin[6], who used three different methods for measuring the electrical conductivity in shock-heated argon: 1) the conventional probe method; 2) interaction between the ionized gas and a magnetic field – which he calls the "magnetic experiment"; 3) rotation of the gas flow and consequent interaction with a magnetic field ("vortex experiment"). The shock Mach range studied was 4–16 and results obtained were in good agreement with Spitzer and Haerm's[7] theory in the shock Mach range 8–10, showing small deviations for higher Mach numbers.

Lamb and Lin[5] measured the electrical conductivity in air for shock Mach numbers 10–18 by the magnetic method and obtained results agreeing with theory.

Petschek and Byron[4] studied the approach to equilibrium in shock Mach range 10–20. They used two different experimental methods, measuring: 1) the intensity of the continuum radiation emitted by the gas; 2) the electrostatical diffusion potential. They succeeded to show that, while the initial stage of the ionization process is dominated by impurities, electron-ion collisions are the rate determining process after the number of electron has reached 10% of its final (equilibrium) value.

Niblet and Blackman[8] investigated ionization in air by a photographic method using a glass tube in which shock waves were produced by discharging a bank of capacitors. They measured the time required for obtaining equilibrium in the shock Mach range 11–17.

The use of microwave methods for measuring electrical properties of ionized gases goes back to the war period. Saenger, Bredt and Goercke[9] in Germany measured the electron density in the exhaust gases of a V-2 rocket by the reflection of centimetric waves, while Andrew, Axford and Sugdon[10] in England found the electrical conductivity of rifle muzzle flames by attenuation measurements in the 3 cm range. Microwave methods were used in the post-war years in investigations of electrical properties of flames[11,12] and of discharges in gases (reviews given by Loeb[13] and Goldstein[14].)

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Low and Manheimer[15, 16] developed a microwave absorption method, by which results on the electrical properties and the ionization rate of shocked gases in Mach range 7–10 were obtained.

The principle of the method is to send a microwave beam through the shocked gas and measure its attenuation as a function of time. Experiments were performed with air, argon and nitrogen using 3 cm waves. For shock Mach numbers 8.2 to 10.2 in air the electron density calculated from attenuation measurements is in good agreement with that predicted from shock-wave theory assuming thermal equilibrium. The most important contribution to the electron density comes from the nitric oxide, NO, whose ionization potential—9.25 ev—is the lowest of all species present. The ionization time, measured from the beginning of the attenuation till its maximum (equilibrium) value is reached, is short (7–12 microseconds). The product of this time by the initial air pressure plotted as function of shock Mach Number extends the results obtained by Niblett and Blackman[8] to a lower Mach range. This points to the following mechanism[17] for NO formations:

$$O_2 \leftrightarrows 2O\,; \quad O + N_2 \leftrightarrows NO + N\,; \quad N + O_2 \leftrightarrows NO + O$$

since relaxation times involved are of the right order of magnitude, according to experimental evidence[18–20].

The experiments with nitrogen were performed with a gas containing 1/4% of oxygen. This small quantity of oxygen was sufficient in order that the dominating factor in the ionization process in the Mach range studied (7.4 to 8.8) should again be nitric oxide. The maximum electron density, calculated from the experiments, was reached in a somewhat longer time (about 15 microseconds) than in air, probably due to the lower oxygen content. Agreement with electron density predicted assuming thermal equilibrium was fair. A rough estimate of the dissociative recombination coefficients of nitric oxide shows order of magnitude agreement with coefficients of typical diatomic gases, quoted by Loeb[13]. An interesting effect, probably due to reaction between the oxygen impurity and the hydrogen driver gas after the contact surface has reached the test section, was observed. This agrees with similar observations by Clouston, Gaydon and Glass[21].

The argon experiments (shock Mach range 7.1–8.5) showed agreement with equilibrium ionization at the lower end of the range only. This seems to be due to the fact that the ionization time of argon (100 microsec. at M = 10, see ref. 4) is longer than test time in the shock tube used. The gas contained 0.2% impurities, which may account for the quick ionization onset and the large spread of values at high Mach numbers. Oscillations in the microwave power output before beginning of the attenuation were observed in the case of argon. This suggests partial omission of 3 cm waves.

The microwave absorption method allows one to obtain information on the electron density and ionization rate in gases heated by medium strength shock waves; in particular relatively low shock Mach numbers, where other known methods gave

unsatisfactory results. may be studied. The attenuation range most suited for this method lies between 0.1 and 10 db; measurements are possible at lower attenuation levels too, providing special precautions to stabilize the oscillator's output are taken. The above mentioned attenuation values may cover a considerable range of state variable of the gas, since it is easy to vary the following parameters within rather wide limits:

a) the effective path of the microwave through the gas, which determines the magnitude of the attenuation coefficient;

b) the microwave frequency;

c) shock strength and initial pressure.

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ON THE METEOROLOGY OF THE PLANET MARS

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Of the constituent gases of the Martian atmosphere the only one which has been definitely identified is CO_2. The presence of water vapour is next to certain, although its amounts are likely to be small. O_2 is also probably present, but again in small amounts. N_2 is likely to be the chief constituent.

Optical methods lead to the estimate that atmospheric pressure at the Martian surface is ca. 85 mb (ca. 64 mm HgO° C), i.e. higher than the pressure at which human blood boils. Atmospheric density at the surface is ca. 10^{-4} g cm^{-3}. Thus pressure and density at the Martian surface are about one order of magnitude smaller than at the Earth's surface. As the Martian g is small (38% of the Earth's g), pressure decreases but slowly with height and at a height of ca. 30 km pressure is the same as that at the same height in the atmosphere of Earth.

The general level of air temperature is about 30°C lower than on Earth. The atmosphere failing to provide an effective greenhouse protection, the daily range of air temperature is nearly 100°C. Under the sub-solar point, temperature reaches about 30°C, while at the winter pole temperature drops to ca. -60°C.

In all probability, the adiabatic temperature lapse rate (g/c_p) is ca. 3.7°C/km for the lower Martian atmosphere. Despite this small value, there is not much evidence in favour of convective cloud formation. Mainly because of the sparsity of cloud cover the planetary albedo is low: 0.15 against the Earth's 0.40. One consequence of the paucity of clouds is that we can often see the solid surface of Mars.

There is some observational evidence that the winds are mainly westerly. The thermal wind equation indicates that the variation of wind with height is moderate, again, mainly because of the small Martian g. On the whole, the dynamics of the Martian atmosphere is governed by the same forces as is the case for the atmosphere of Earth.

HEAT TRANSFER STABILITY ANALYSIS OF SOLID PROPELLANT ROCKET MOTORS

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This paper presents a theoretical investigation of the causes of unstable burning in solid propellant rocket motors. A possible mechanism of unstable burning is developed. Small perturbation analysis is applied to a set of differential equations describing heat transfer in the solid as well as in the gas film near the burning surface. Criteria of instability are derived directly from the equations having only the physical properties of the propellant as parameters. The assumption of a time lag, necessitated in other theories, is avoided. Thus, a clearer physical picture of the instability situation is obtained and verification by experimental tests made feasible. It is shown why an endothermic reaction at the decomposing surface of the solid propellant favours stability. The stabilizing influence of turbulators and radial holes drilled into the charge is explained.

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